SEOUL NATIONAL UNIVERSITY
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

SYSTEM CONTROL **Fall 2014**

Midterm Exam Solution **Date: October 21, 2014 (Tue)**
Closed book, closed note **11:00-12:10**

Student ID: ____________________ Name:__________________

Problem	Points
1(15)	
2(10)	
3(10)	
4(15)	
5(15)	
Total (65)	

[1] (15 points) Describe followings:

(1) linear dynamic systems

Systems : combination of components acting together to perform specific objectives
Dynamic systems: output depends on all the past input applied to the system
Input output relationships: Differential equation
Static systems: output depends on the current input applied to the system
Input output relationships: algebraic equation

각 요소들 설명 생략 시 -1
요소 설명 모두 생략 시 정확한 추가 설명 없으면 -2 점

Linear systems : principle of superposition

(2) Control system
….
Control : Applying inputs to the system to correct or limit deviation of the output values from desired values
Systems : combination of components acting together to perform specific objectives

위 정의에 의미가 부합하면 5
Control 의미 부합하지 않으면 -1
1 에 시스템 설명 있으면 생략 가능
없으면 -1
입출력 신호 언급없으면 -1

(3) Stability

설명 틀리면 0 점
Equilibrium point 로 수렴 빠지면 -1
선형 시스템 안정성 설명 빠지면 -1

[2] (10 points) Obtain the condition of K for stabilizing the following system using Routh's stability criterion.

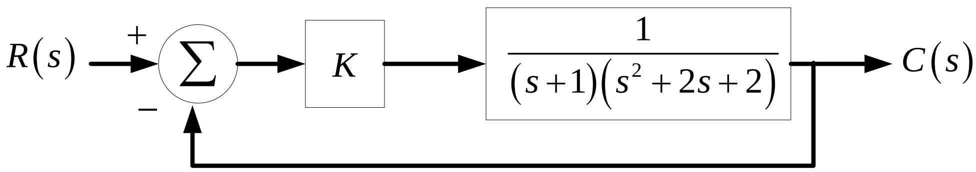

Fig.2 Feedback control system

$$charac.eqn = s^3 + 3s^2 + 4s + 2 + K = 0$$

$$\begin{array}{lcc}
s^3 & 1 & 4 \\
s^2 & 3 & 2+K \\
s^1 & \dfrac{10-K}{3} & 0 \\
s^0 & 2+K &
\end{array}$$

$$\therefore -2 < K < 10$$

+ 4	Characteristic Equation 까지
+ 10 (Routh's stability criterion 이용)	정답
과정 계산 실수 시 -1	
정답 계산 실수 시 -2	
범위 실수 시 -2	
범위 추가 시 -1	

[3] (10 points) Obtain the steady state values of the following equations if the values exist.

(1) $Y(s)=\dfrac{4}{s(s+1)(s+2)(s+3)}$

$$Y(\infty)=\lim_{s\to 0} sY(s)=\lim_{s\to 0}\frac{4}{(s+1)(s+2)(s+3)}=\frac{2}{3}$$

$$y(t)=-\frac{2}{3}e^{-3t}+2e^{-2t}-2e^{-t}+\frac{2}{3}$$

(2) $Y(s)=\dfrac{3}{s(s+1)^2(s-2)}$

The final value theorem is not applicable because Y(s) has a pole s = 2.

$$y(t)=(t+\frac{4}{3})e^{-t}+\frac{1}{6}e^{2t}-\frac{3}{2}$$

	(1)	(2)
+ 3	FVT or Inverse Laplace Transform	Inverse Laplace Transform
+ 5	정답	정답
	계산 실수 시 -1	FVT 적용시 -2, 계산 실수 시 -1

[4] (15 points) Consider a system shown in the Fig.4 below.

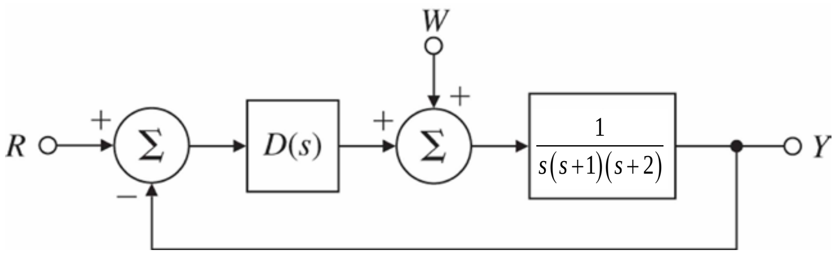

Fig.4 Feedback control system in the presence of disturbance

모든 문제에 대해 계산 실수 시 -1

(1) When $D(s)=K$, i.e., proportional control, obtain the transfer function $G_W(s)=\dfrac{Y(s)}{W(s)}$.

$$D(s)=K$$

$$\frac{Y(s)}{W(s)}=\frac{\dfrac{1}{s(s+1)(s+2)}}{1+K\dfrac{1}{s(s+1)(s+2)}}=\frac{1}{s^3+3s^2+2s+K} \qquad +4$$

(2) Obtain the steady state output for the unit step disturbance, i.e., $w(t)=1$.

$$w(t)=1\rightarrow W(s)=\frac{1}{s} \qquad +2$$

$$y(t)=\lim_{t\to\infty}=\lim_{s\to 0}sY(s)=\lim_{s\to 0}\frac{1}{s^3+3s^2+2s+K}=\frac{1}{K} \qquad +2$$

(3) Design a PI controller such that the steady state output for the unit step disturbance is zero.

$$D(s)=K\left(1+\frac{1}{T_i s}\right)=K_P+K_I\frac{1}{s} \qquad +1$$

$$\frac{Y(s)}{W(s)}=\frac{\dfrac{1}{s(s+1)(s+2)}}{1+\left(K_P+\dfrac{1}{s}K_I\right)\dfrac{1}{s(s+1)(s+2)}}=\frac{1}{s^3+3s^2+2s+K_P+K_I\dfrac{1}{s}}$$

$$=\frac{s}{s^4+3s^3+2s^2+K_Ps+K_I} \qquad +2$$

$$w(t)=1\rightarrow W(s)=\frac{1}{s} \qquad +2$$

$$y(t)=\lim_{t\to\infty}=\lim_{s\to 0}sY(s)=\lim_{s\to 0}s\frac{s}{s^4+3s^3+2s^2+K_Ps+K_I}\frac{1}{s}=0 \qquad +2$$

계산 실수나 오류가 있으나 나머지 계산이 모두 맞으면 +3

위의 풀이와 전혀 다른 방식으로 풀었으나 논리가 맞으면 +3

[5] (15 points) Consider a feedback control systems in Fig.5-(a). The root loci of the system are shown in Fig.5-(b).

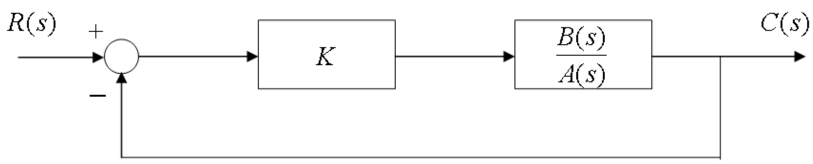

Fig.5-(a) Feedback control systems

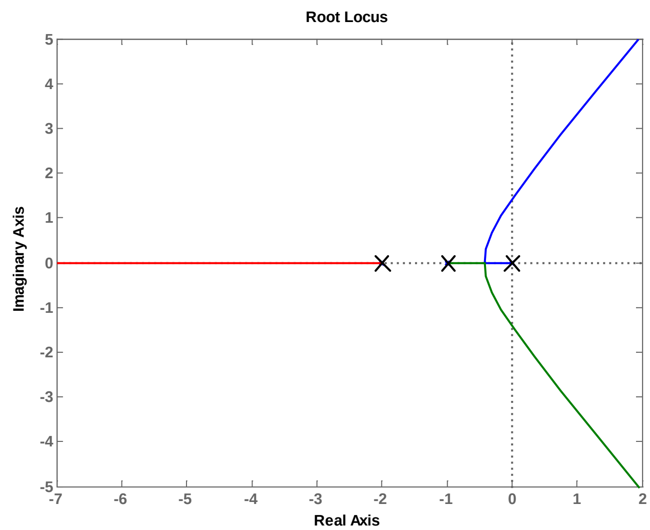

Fig.5-(b) Root locus of the feedback control systems

(1) Obtain the open-loop transfer function of the system, $\frac{B(s)}{A(s)}$, where $\left|\frac{B(1)}{A(1)}\right| = \frac{1}{6}$.

$$\frac{B(s)}{A(s)} = \frac{k}{s(s+1)(s+2)}$$

$$\left|\frac{B(1)}{A(1)}\right| = \frac{k}{|1||(1+1)|(|1+2)|} = k\frac{1}{6} = \frac{1}{6}$$

$$\therefore k = 1$$

(2) Determine the angles of asymptotes of the root loci (as s approaches infinity).

$$\lim_{s\to\infty} G(s) = \lim_{s\to\infty} \frac{\mathrm{K}}{s(s+1)(s+2)}$$

$$= \lim_{s\to\infty} \frac{\mathrm{K}}{s^3}$$

$$-3\angle s = \pm 180°(2k+1) \qquad k = 0,1,2,\cdots$$

$$Angles\ \ of\ \ asymptotes = \frac{\pm 180°(2k+1)}{3} \qquad k = 0,1,2,\cdots$$

(3) Determine the proportional gain, K, where the root loci across the imaginary axis **by use of Routh's stability criterion.**

Since the characteristic equation for the present system is

$$s^3 + 3s^2 + 2s + K$$

The Routh array becomes

$$\begin{array}{lcc} s^3 & 1 & 2 \\ s^2 & 3 & K \\ s^1 & \dfrac{6-K}{3} & \\ s^0 & K & \end{array}$$

The value of K that makes the s^1 term in the first column equal zero is $K = 6$.