SEOUL NATIONAL UNIVERSITY
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

SYSTEM CONTROL	**Fall 2014**
Final Exam	**Date: December 11, 2014 (Th)**
Closed book, closed note	**11:00-12:30 am**

Student ID: ____________________ Name:__________________

Problem	Points
1(20)	
2(15)	
3(15)	
4(20)	
5(15)	
6(10)	
7(20)	
Total (115)	

[1] (20 points) Describe followings:

(1) Linear Control Systems

Solution

Systems: combination of components working together

Control : Applying inputs to the system to correct or limit deviation of the output values from desired values

Linear superposition

(+1)Open-loop/closed- loop control systems
(+1)feedback control system Block diagram

(2) Stability

Definition: equilibrium, linear asymptotic stability
Linear system stability: Re(Pole)<0

Nyquist stability criterion (+1)

(3) controllability and observability

Definition : state transfer
State equation
State feedback, controllability condition, controllability matrix
State-feedback, regulator pole (A-BK)
(+1) separation property

Definition: observability
Observer equation
Observability matrix
Observer pole (A-LC)

(4) State, state equation, state space

State x:
The smallest set of variables such that knowledge of these variables at t=t0 , together with the knowledge of the input for t>t0 , completely determines the behavior of the system at any time t>t0.

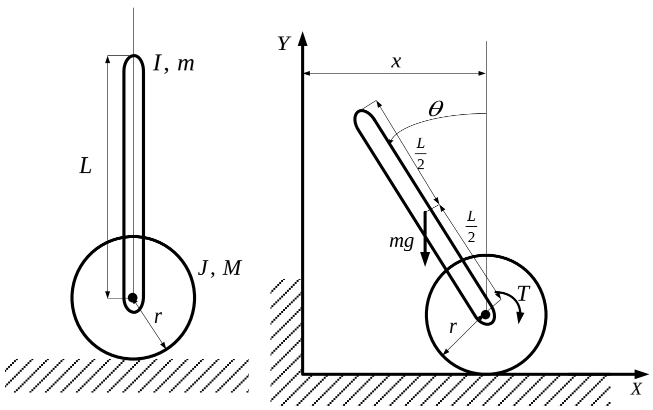
State equation

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

State space:
The n-dimensional space, whose coordinate axes consist of the x_1 axis, x_2 axis, …, x_n axis is called a state space. Any state can be represented by a point in the state space

[2] An inverted bar mounted on a motor-driven cart is shown in Figure below. The objective of the control problem is to transfer the position of the cart to any desired place and to keep the bar in a vertical position. The inverted bar is unstable in that it may fall over any time in any direction unless a suitable control force is applied. Here we consider only a two-dimensional problem in which the pendulum moves only in the X-Y plane. The control force F is applied at the cart. Assume that the center of gravity of the bar is at its geometric center.



$$I\ddot{\theta} = m\ddot{x}\frac{L}{2}\cos\theta - m\left(\frac{L}{2}\right)^2\ddot{\theta} + mg\frac{L}{2}\sin\theta$$

The velocity of the cart can be controlled by a motor and the motor angular speed can be represented as

$$\omega(s) = \frac{k}{1+\tau s}u(s)$$

Where

$$\omega(t) = \frac{d\theta_M(t)}{dt},$$

$x(t) = r \cdot \theta_M(t)$, and $u(t)$ is the control input.

(1) Define the state as follows:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix}$$

If we consider θ and x as the outputs of the system, then

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \theta \\ x \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$$

Obtain a state-space model of the system. The model is of nonlinear dynamics. And then obtain a linear state-space model assuming that the θ is small.

(2) Assume that the measurements of $x_1(t), x_2(t),$ and $x_4(t)$ are available. Therefore, you can use a control law of following form:

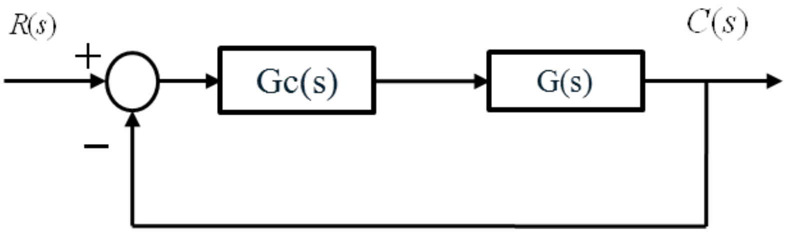
$$u(t) = -k_1 \cdot x_1(t) - k_2 \cdot x_2(t) - k_4 \cdot x_4(t)$$

Using the linearized model obtained in problem (2), determine the gains, $k_1(t), k_2(t),$ and $k_4(t)$ so that $x_1(t)$ tends to zero as t increases, i.e.,

$$\lim_{t \to \infty} x_1(t) = 0 .$$

(3) It is possible to have both $x_1(t)$ and $x_2(t)$ tends to zero by the control law of the form in the problem (2). What happens, in this case, to the position of the system? Analyze the behavior of the system.

[3] (15 points) Consider a unity feedback closed-loop control system shown below



When the controller is proportional control, i.e.,
$G_c(s) = K = 10$

The Bode diagram of the system is shown in Figure 5.

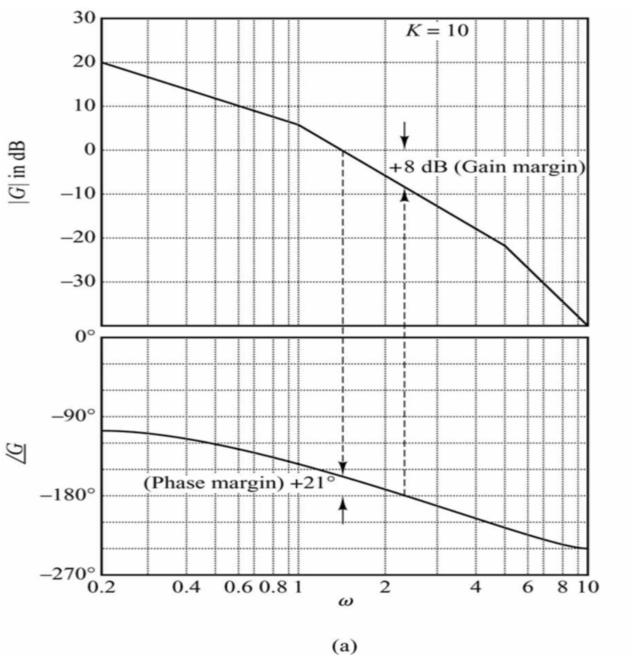


Fig. 5 Bode diagram of the system

(1) Sketch the Nyquist plot of the system.
Solution:

(2) Determine the stability of the system using Nyquist Stability Criterion.

Solution: stable

(3) When the control gain is increased from 10 to 100, obtain the phase and gain margins of the system.

Solution:

Phase margin: -30 degree
Gain margin : -12 dB

[4] (20 points) The input u, and the output y, of the single-input, single-output system are related by

$$\frac{d^3}{dt^3}y(t)+6\frac{d^2}{dt^2}y(t)+2\frac{d}{dt}y(t)+3y(t)=2\frac{d^2}{dt^2}u(t)-5\frac{d}{dt}u(t)-5u(t)$$

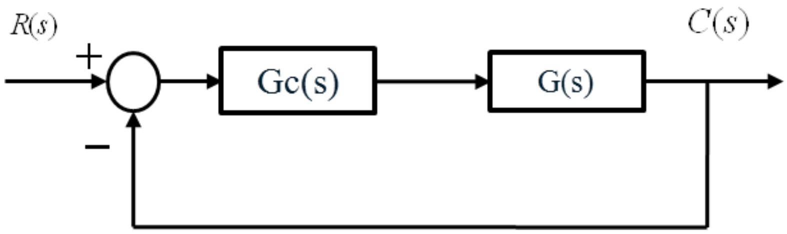
(1) Find the transfer function from U to Y.

(2) Is this system stable?

(3) If u(t)=2 for all $t \geq 0$, what is the steady state value of y(t)?

(4) Suppose that the input is sinusoidal, $u(t) = \sin(10t)$. What is the steady state amplitude of the output y(t)?

[5] (15 points) Consider a system shown below.



$G_c(s) = K,$

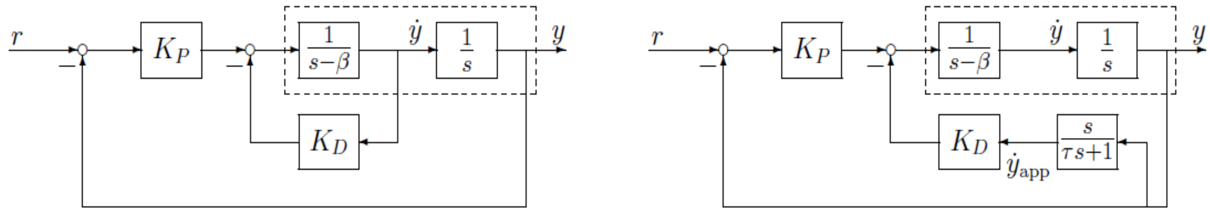
$G(s) = \frac{1}{s}$

(1) Under what conditions is the system stable?

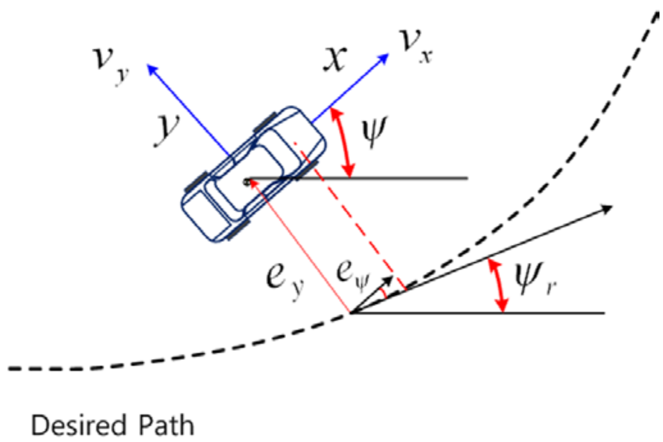
(2) If the system is stable, what is the time constant?

(3) If the system is stable what is the steady-state gain from r to y ?

[6] (10 points) Block diagram for two systems are shown below. The constant β is positive, $\beta > 0$. The system is stable if and only if $K_p > 0$ and $K_D > \beta$. What are the conditions on K_p, K_D and τ such that the system on the right is stable?



[7] (20points) Consider an autonomous vehicle system shown in the Figure below:



The error dynamics are represented as follows:

$$\frac{de_y}{dt} = v_y + v_x \sin e_\psi - e_y \gamma \sin e_\psi$$
$$\frac{de_\psi}{dt} = \frac{d\psi}{dt} - \frac{d\psi_r}{dt} = \gamma - \frac{d\psi_r}{dt} = \gamma + w$$
$$e_\psi = \psi - \psi_r$$
$$w(t) = -\frac{d\psi_r}{dt} = -\frac{v_x}{R}$$

Where v_x is the vehicle speed and R is the radius of the path. The vehicle yaw rate, γ, is proportional to the steering input, δ, i.e.,

$$\frac{d\gamma(t)}{dt} = K_\gamma \delta(t)$$

(1) Assuming that the vehicle speed, v_x, is constant and the vehicle lateral velocity, v_y, is very small, obtain a linear state equation of the steering control system. Here, define the state and input as follows:

$$x_1 = e_y$$
$$x_2 = e_\psi$$
$$y(t) = x_1(t)$$
$$u(t) = \delta(t)$$

(2) Is the state equation model obtained in (1) controllable?

Solution: controllable C=[B AB] Rank [C] =2

(3) Design a state feedback controller such that $\begin{matrix} x_1(t) \to 0 \\ x_2(t) \to 0 \end{matrix}$ as the time, t, increases when the disturbance, w, is zero.

(4) For the controller designed in (3), compute the steady state output error when the disturbance, w, is constant.

(5) Design a steering controller such that the steady state output error tends to zero when the disturbance, w, is constant.