1.(20pts) Consider an electron trapped in a one-dimensional box with infinitely high potential. The box extends from $x = 0$ to $x = a$. The electron has the mass m and the total energy E.

(a) (10ts) Find the energy eigenvalue E_n of the electron by solving Schrodinger equation,

(b) (10pts) Find the eigenfunction ψ_n corresponding to E_n. Also, find the probability density that the electron can be found between x = 0 and $a/3$ for the second excited state ($n = 3$).

2. (25pts) Answer the following questions.

(a) (10pts) Describe the energy for a free electron, a strongly bound electron, and an electron in a periodic potential (i.e., in a crystal), respectively. Why do we get these different band schemes?

(b) (10pts) According to the Kronig-Penny model of one dimensional periodic potential distribution, Schrödinger equations using Bloch function as the electron wave function in the crystal have solutions if the following relation is satisfied;

$$P\frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka \qquad P = \frac{maV_0 b}{\hbar^2}$$

(c) (5pts) Using this relation, explain why forbidden energy bands are formed in the crystal.

3. (20pts) Answer the following questions.

(a) (5pts) Calculate how much the kinetic energy of a free electron at the corner of the first Brillouin zone of a simple cubic lattice (three dimensions!) is larger than that of an electron at the midpoint of the face.

(b) (5pts) Calculate the main lattice vectors in the reciprocal space of an fcc crystal.

(c) (10pts) Using the Drude postulation, derive the conductivity σ as a function of N_f (# of free electron per unit volume), electron charge e, electron mass m, and relaxation time τ. Decribe the temperature dependence of the electrical resistivity in pure metal on the basis of Drude conduction theory. Also, explain briefly why Drude conduction theory needs a modification and how it is modified by quantum mechanical consideration.

4. (20pts) Answer the following questions

(a) (10pts) Derive the effective mass, m^* given by

$$m^* = \hbar^2 \left(\frac{d^2E}{dk^2}\right)^{-1}$$

(b).(10pts) Consider a semiconductor with 10^{13} donors/cm^3 which has a binding energy of 10 meV. What is the concentration of extrinsic conduction electrons at 300 K ? Also, assuming a gap energy of 1 eV (and $m^* = m_0$) at 300 K, what is the concentration of intrinsic conduction electrons? Compare the concentration of extrinsic conduction electrons with that of intrinsic ones.

5. (15pts) Answer the following questions related to semiconductors.

(a) (5pts) An *n*-type semiconductor is brought to contact with a metal. If the work function of the metal(ϕ_M) is larger than that of the semiconductor (ϕ_S), the contact shows a rectifying (Schottky barrier) behavior. Explain the reason for it by sketching the band diagram *before* and *after* contact.

(b) (10pts) Draw a p-n junction in equilibrium, and explain the operation of p-n rectifier (diode) by drawing the band diagram modifications for forward and reverse bias.