Electronic Properties of Materials **Final Exam** Dec. 15 (Mon), 2014

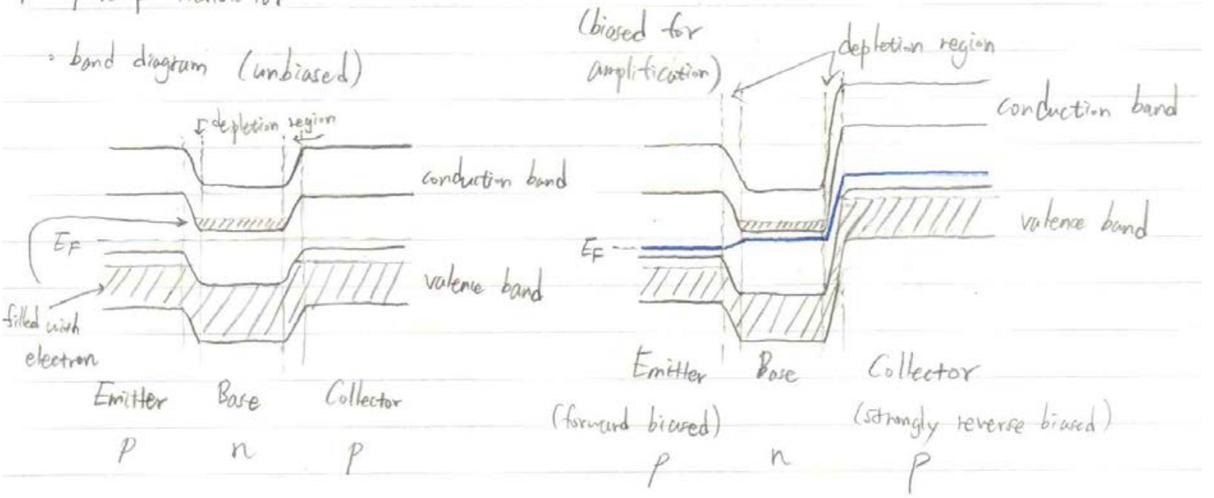
(Prof. Sang-Im Yoo)

1. (20pts) Answer the following questions.

(a) (10pts) Describe the band diagram and function of a *p*-*n*-*p* transistor.

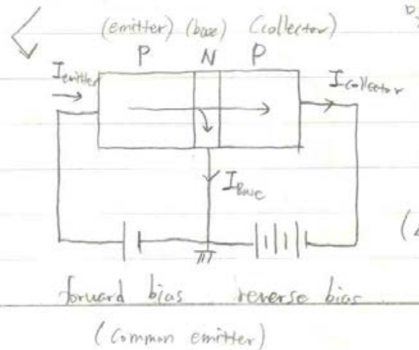
4 p-n-p transistor

• band diagram (unbiased)



npn transistor 와 비교했을 때 major charge carrier 가 hole 이라는 점이 다르다.

우선 emitter 에 존재하는 hole 이 potential barrier 를 뛰어넘게 되면 base 지역에 도달하게 되고, 이 때 일부분이 base의 전자와 결합하여 소멸된다. 이러한 소수의 전자는 전원의 -극에서 계속 공급되므로 이것이 약간의 base current 가 된다. emitter 에서 base 로 온 hole 중 전자와 결합하지 않은 것은 강한 reverse biase 에 의해 collector 쪽으로 가서 collector current 를 형성한다. 또한 emitter 의 hole 은 전원의 + 극에서 점차 보급되어 emitter current 를 형성한다. 따라서 emitter current 의 대부분은 collector current 가 되고, 일부분만 base current 를 형성하게 된다. 위와 같은 transistor에서 base current 의 작은 변화로 collector current 의 매우 큰 변화를 얻을 수 있으며, 이를 amplification 에 이용한다. 또한 base voltage 의 조절에 의해 emitter 에서 collector 로의 전류를 막는 electronic switch 기능도 나타낼 수 있다.



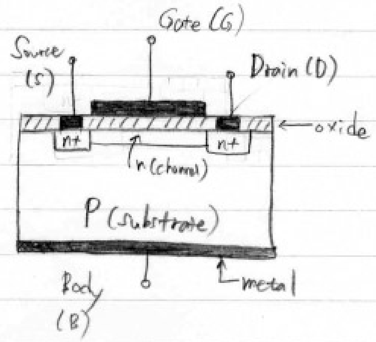
(common emitter)

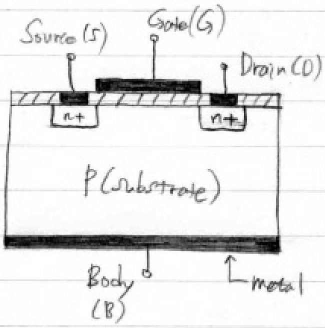
$$I_{emitter} = I_{Base} + I_{Collector}$$

$$(\Delta I_{emitter} = \Delta I_{Base} + \Delta I_{Collector})$$

(b) (10pts) Schematically draw the depletion-type (or normally-on) MOSFET and the enhancement-type(or normally-off) MOSFET, respectively. Briefly explain their operation principles.

2.(c) depletion-type MOSFET (normally-on)　　　　enhancement-type MOSFET (normally off)





normally on: channel 부분에 n-type doping 이 되어있어 Gate 와 Body 사이에 전압이 걸리지 않더라도 S-D 사이에 전류가 흐른다. G-B 사이에 negative 전압이 걸리면, 전자가 channel 영역에서 substrate 로 밀려나게 되고, 결국 특정 전압 (-) 이하에서는 S-D 사이에 전류가 흐르지 않게 된다.

normally off: P-type doped substrate 가 그대로 channel 부분에 있어서 G-B 사이에 전압이 걸리지 않으면 S-D 사이에도 전류가 흐르지 않는다. G-B 사이에 positive 전압이 걸리면, 대부분의 hole 은 body 부근으로 가게 되고, negative charge carrier 가 channel 부근에 모이게 된다 따라서 특정 전압 (+) 이상에서는 S-D 사이에 전류가 흐르게 된다.

2. (15pts) Answer the following questions.

(a) (10pts) Show the ionic conductivity in ionic conductors is given by

$$\sigma_{\text{ion}} = \sigma_0 \exp\left[-\left(\frac{Q}{k_B T}\right)\right] \qquad \sigma_0 = \frac{N_{\text{ion}} e^2 D_0}{k_B T}$$

(a) Einstein relation, $\mu_{ion} = \frac{De}{k_B T}$, $\sigma_{ion} = N_{ion} e \mu_{ion}$, $D = D_0 \exp\left[-\frac{Q}{k_B T}\right]$

Arrhenius equation

$\sigma_{ion} = N_{ion}\, e \cdot \frac{De}{k_B T}$

$= N_{ion}\, e^2 \frac{D_0}{k_B T} \exp\left[-\frac{Q}{k_B T}\right]$

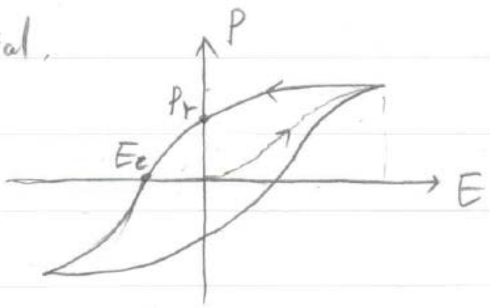
Let $\sigma_0 = \frac{N_{ion} e^2 D}{k_B T}$　　$\therefore \sigma_{ion} = \sigma_0 \exp\left[-\frac{Q}{k_B T}\right]$

(b) (b) (5pts) Show that ***P*** = *(ε-1)ε₀* ***E*** in dielectric materials, where ***P*** and ***E*** are the dielectric polarization and electric field, respectively. Also, schematically draw a hysteresis loop for a ferroelectric material on *P*-*E* diagram. Please express a remanent polarization (P_r) and a coercive field (E_c) on the diagram.

$$\begin{cases} D = \varepsilon\varepsilon_0 E \\ D = \varepsilon_0 E + P \end{cases} \rightarrow \varepsilon\varepsilon_0 E = \varepsilon_0 E + P \quad \therefore P = (\varepsilon - 1)\varepsilon_0 E$$

for vacuum, $\varepsilon = 1$, $\therefore P = 0$, $D = \varepsilon_0 E$

for a ferroelectric material,



3. (30pts) Answer the following questions.

(a) (5pts) What are the definitions and physical meaning of the refraction index *n* and the damping constant *k*, respectively?

(a) ① $n = C_{vac}/C_{med} = \frac{c}{v}$ (definition)

빛이 medium 을 지날 때 굴절되는 정도, medium 에서 빛의 속력이 줄어드는 정도를 나타내는 parameter 이다. (physical meaning)

② $\hat{n} = n - ik = \frac{K \text{ (wave number in medium)}}{K_0 \text{ (wave number in vacuum)}} \rightarrow K = K_1 - iK_2 \quad \therefore n = \frac{K_1}{K_0},\ k = \frac{K_2}{K_0}$ (definition)

(in the textbook)

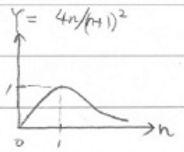
In the case of a plane-polarized wave that propagates along the positive z-axis and which vibrate in the x-direction

↳ $\mathcal{E}_x = \mathcal{E}_0 \exp\left(i\omega\left(t - \frac{zn}{c}\right)\right)$, $(\hat{n})^2 = \varepsilon - \frac{\sigma}{2\pi\varepsilon_0 \nu} i$

이 때, $\hat{n} = n - ik$ 에서의 k는 damping constant 라 한다.

(b) (5pts) Express the reflectivity *R* as a function of refractive index *n* and damping constant *k*. On the basis of this relationship explain why ceramics and polymers (n ~ 1.5, $k \sim 10^{-7}$) exhibit very low *R* while metals (n < 1, $k > 3$) are good reflectors in the infrared region.

4. (a) $R = \dfrac{(n-1)^2+k^2}{(n+1)^2+k^2}$, ceramic 과 polymer 에서는 통상적으로 $n \sim 1.5$, $k \sim 10^{-7}$ 값을 가진다. 이 때 $R \simeq (n-1)^2/(n+1)^2 = 1 - 4n/(n+1)^2$ 로 나타낼 수 있다



왼쪽 그림과 같이 Y 값이 변하므로, R 값 $(R = 1 - Y)$은 $n=1$ 근방에서 0 가 된다.

ceramic, polymer 의 $n \sim 1.5$ 이므로 R은 매우 작은 값을 가지게 된다.

반면 metal 은 n에 비해 k 값이 크게 되는데 $(n<1,\ k>3)$ IR region 의 경우 이 차이가 더 심해져 $R = \dfrac{(n-1)^2+k^2}{(n+1)^2+k^2} \approx \dfrac{k^2}{k^2} = 1$ 에 근접하게 된다

(c) (5pts) Derive the Hagen-Rubens relation from the next equation.

$$R = \frac{\sqrt{\varepsilon_1^2+\varepsilon_2^2}+1-\sqrt{2(\sqrt{\varepsilon_1^2+\varepsilon_2^2}+\varepsilon_1)}}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}+1+\sqrt{2(\sqrt{\varepsilon_1^2+\varepsilon_2^2}+\varepsilon_1)}}$$

(c) $R = \dfrac{\sqrt{\varepsilon_1^2+\varepsilon_2^2}+1-\sqrt{2(\sqrt{\varepsilon_1^2+\varepsilon_2^2}+\varepsilon_1)}}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}+1+\sqrt{2(\sqrt{\varepsilon_1^2+\varepsilon_2^2}+\varepsilon_1)}}$ Hagen-Ruben relation 을 유도하기 위해서는 metal, IR region 으로의 가정이 필요하다. $(\varepsilon_2^2 \gg \varepsilon_1^2)$

$$\therefore R \doteqdot \frac{\varepsilon_2+1-2\sqrt{\varepsilon_2}}{\varepsilon_2+1+2\sqrt{\varepsilon_2}} = 1 - \frac{2\sqrt{\varepsilon_2}}{\varepsilon_2+1+2\sqrt{\varepsilon_2}} = 1 - \frac{2\sqrt{2}}{\sqrt{\varepsilon_2}+\cancel{\frac{1}{\sqrt{\varepsilon_2}}}+\cancel{\sqrt{2}}} \quad (\because \text{IR region},\ \varepsilon_2 \approx 10^4)$$

$$\therefore R = 1 - \frac{2\sqrt{2}}{\varepsilon_2} \underset{(\varepsilon_2 = \sigma/2\pi\varepsilon_0\nu)}{=} 1 - 4\sqrt{\frac{\pi\varepsilon_0\nu}{\sigma}} \underset{(\text{for small frequency},\ \sigma=\sigma_0)}{=} 1 - 4\sqrt{\frac{\pi\varepsilon_0\nu}{\sigma_0}}$$

(d) (5pts) In order to interpret absorption band (absorption of light into materials at high frequency), Lorentz postulated that the electrons are bound to their respective nuclei. Construct the equation of motion for the electron in Lorentz model and explain its physical meaning. Also compare it with free electrons without damping model.

(d) Lorentz model : nuclei 에 bound 된 electron 고려

$$m\frac{d^2x}{dt^2} = eE_0 \exp(i\omega t) - \gamma'\frac{dx}{dt} - Kx$$

$eE_0 \exp(i\omega t)$: external electric field with force eE

$\gamma'\frac{dx}{dt}$: electron 이 oscillation 할 때 radiation 으로 잃어버리는 energy 와 관련된 term

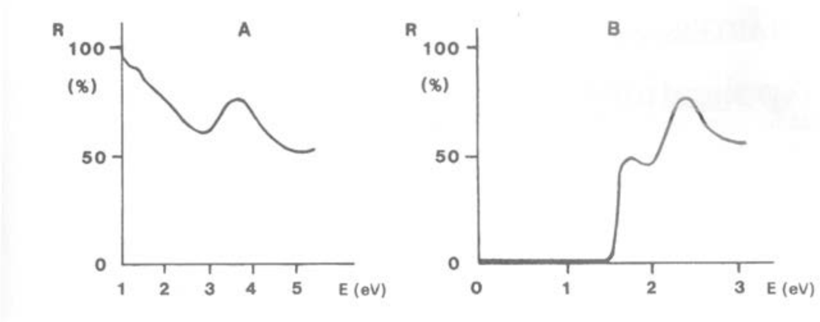
Kx: nuclei 와 electron 의 결합정도를 나타내는 term

free electrons without damping

$$m\frac{d^2x}{dt^2} = eE_0 \exp(i\omega t)$$

; Lorentz model 과 비교했을 때, damping term 등은 모두 사라지고 오직 전자에 작용하는 external E-field 만을 고려한다.

(e) (10pts) Below the reflection spectra for two materials A and B are given. What type of material belongs to reflection spectrum A, what type to B? (Justify). Note the scale difference! Also, for which of the materials would you expect intraband transitions in the infrared region? Explain the reason.



A: metal (high reflectivity in IR region, intraband transition)
B: Semiconductor (Low reflectivity in IR region, band gap 존재)
intraband transition 이 일어날 수 있는 물질은 A 이다 (in IR region)
이는 metal 의 경우 conduction band 에 비어있는 부분이 항상 존재하기 때문에
낮은 energy (IR) 으로 band 내에서의 transition 이 가능하다

4. (40pts) Answer the following questions.

(a) (10pts) Using a universal relation between magnetic field $\boldsymbol{H}$, magnetic induction $\boldsymbol{B}$ and magnetization $\boldsymbol{M}$, show the relationship between the relative permeability μ_r and the susceptibility χ in SI unit. Explain how this relationship changes in cgs (Gaussian) unit.

<SI>

$B = \mu_r \mu_0 H$

$B = \mu_0 (H+M)$, $M = \chi H$ → $B = \mu_0 (1+\chi) H$ → $\mu = 1+\chi$

<cgs>

$B = \mu_r H$

$B = H + 4\pi M$, $M = \chi H$ → $B = (1+4\pi\chi) H$ → $\mu_r = 1+4\pi\chi$

(b) (5pts) What are the relative permeability μ_r and the susceptibility χ (in SI unit) of a superconductor?

for a superconductor : $\mu_r = 0$, $\chi = -1$

(c) (5pts) Using the Langevin electron orbit theory for paramagnetism given below,

$$\boldsymbol{M} = \boldsymbol{n\mu_m}\left(\boldsymbol{coth\xi} - \frac{\boldsymbol{1}}{\boldsymbol{\xi}}\right) = \boldsymbol{n\mu_m}\left(\frac{\boldsymbol{\xi}}{\boldsymbol{3}} - \frac{\boldsymbol{\xi^3}}{\boldsymbol{45}} + \frac{\boldsymbol{2\xi^5}}{\boldsymbol{945}} - \cdots\right)$$

derive the curie law for the paramagnetic materials.

$M_0 = n\mu_m$: the maximum possible magnetization

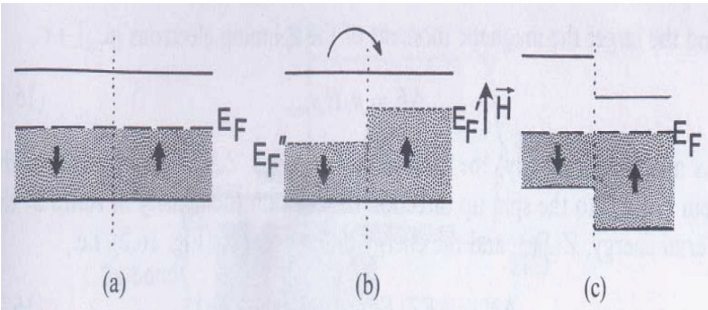
Because $\zeta = \dfrac{\mu_m \mu_0 H}{k_B T}$ is usually small,

$$\frac{M}{M_0} = \frac{\zeta}{3} \Rightarrow \therefore M = M_0 \frac{\zeta}{3} = \frac{n\mu_m^2 \mu_0 H}{3k_B T}$$

$$\chi_{para} = \frac{M}{H} = \frac{n\mu_m^2 \mu_0}{3k_B}\frac{1}{T} \equiv C \cdot \frac{1}{T}$$

$$C = \frac{n\mu_m^2 \mu_0}{3k_B}$$

(d) (5pts) Describe the temperature dependency of susceptibility for paramagnetic metals, and explain its behavior quantum mechanically.



paramagnetic metals의 susceptibilities는 에너지 이론에 기초한다
스핀 전자들의 Magnetic moment가 paramagnetism에 크게 기여하므로 온도에 크게 의존하지 않는다.

(e) (5pts) The basic unit for the magnetic moment is the Bohr magneton, μ_B. Show that 1 μ_B is equal to *eh/(4πm)* by using the Bohr model. Where *e* is the electron charge, *m* is the electron mass and *h* is the Plank constant.

$$\boxed{\mu_m = I \times A = \frac{e}{t}A = \frac{e}{s/v}A = \frac{ev\pi r^2}{2\pi r} = \frac{evr}{2} \quad (A\text{= area of loop})}$$

$$2\pi r = n\lambda = n\frac{h}{p} \rightarrow \quad rp = \frac{h}{2\pi}n = \hbar n, \ (mvr = \text{angular momentum})$$

$$mvr = \hbar n = \frac{nh}{2\pi} \qquad (16.9)$$

$$\boxed{\mu_m = \frac{enh}{4\pi m}} \quad (16.10) \qquad \textbf{For } n = 1, \quad \mu_m = \frac{eh}{4\pi m} \quad (16.11)$$

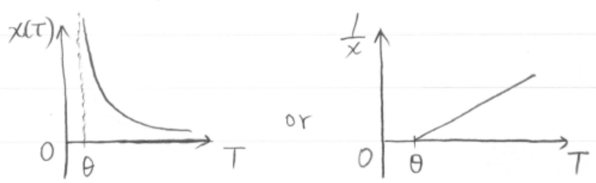
$$\mu_B = \frac{eh}{4\pi m} = 9.274 \times 10^{-24} \left(\frac{J}{T}\right) \qquad (16.12) \qquad \textbf{Bohr magneton}$$

(f) (5pts) Draw and explain the $\chi(T)$ curves for normal diamagnets and non-metallic paramagnets following the Curie-Weiss law.

· normal diamagnets

· normal diamagnets 에서 χ는 음수 값을 가지며, χ는 보통 $-10^{-6} \sim -10^{-3}$ 의 작은 값을 갖는다. 즉 외부 자기장에 반대 방향으로 자화가 일어나며, 자화의 정도는 온도에 거의 의존하지 않는다.

· normal paramagnets



normal paramagnets 은 Curie-Weiss law 를 따르며. 이 때 χ와 T의 관계는

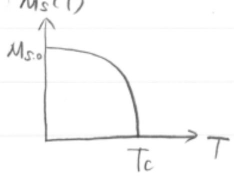
$\chi = \dfrac{C}{T-\theta}$ (C : Curie const) 로 나타난다

외부에 가해준 자기장과 같은 방향으로 자화가 일어나며, 온도가 올라 갈수록 spin 배열이 어긋나는 경향에 따라 χ는 감소하게 된다.

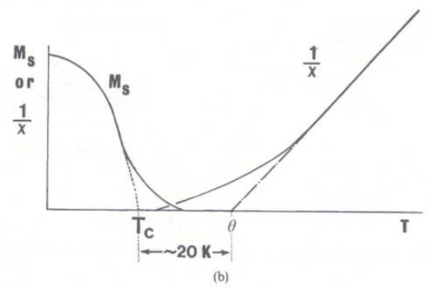
· anti ferro magnets

(g) (5pts) Draw and explain the $M(T)$ curve below Curie temperature T_c and $\chi(T)$ curve above T_c for ferromagnets.

· normal ferromagnets



ferromagnets 에서는 magnetic domain 들의 정렬을 통해 자화가 일어나며, 하나의 온도에서 field를 가해주었을 때 재료가 최대로 자화될수 있는 saturation magnetization (M_s) 값이 존재하게 된다. 이 M_s는 온도가 높아짐에 따라 작아지게 되고, T_c 이상에서는 magnetic domain 자체가 사라지게 된다.



(b)

5. (15pts) Answer the following questions.

(a) (10pts) Derive the empirical Dulong-Petit law (molar heat capacity, C_v ~3R, R=gas constant) for the heat capacity of materials using classical theory. If temperature is decreased to a very low temperature, materials exhibit a reduction in their values of heat capacity. Explain this behavior using the Einstein model qualitatively.

The average energy of the oscillator is

$$E = k_B T.$$

The average energy per atom is

$$E = 3k_B T \ \ (3-\text{dimensional}).$$

The average kinetic energy of a particle is

$$E = \frac{3}{2} k_B T.$$

The total energy of a vibrating lattice atom is thus

$$E = 2 \cdot \frac{3}{2} k_B T. \text{ (kinetic energy + potential energy)}$$

Then the total internal energy per mole is

$$E = 3N_0 k_B T = 3RT$$

Finally, the molar heat capacity is

$$C_V = \left(\frac{\partial E}{\partial T} \right)_V = 3R.$$

Heat capacity at a constant volume

$$C_v = \left(\frac{\partial U'}{\partial T}\right)_v = 3nh\nu(e^{h\nu/kT}-1)^{-2}\frac{h\nu}{kT^2}e^{h\nu/kT}$$
$$= 3nk\left(\frac{h\nu}{kT}\right)^2\frac{e^{h\nu/kT}}{(e^{h\nu/kT}-1)^2}$$

Einstein temperature is

$$\theta_E = \frac{h\nu}{k}$$

So,

$$c_v = 3R\left(\frac{\theta_E}{T}\right)^2\frac{e^{\theta_E/T}}{(e^{\theta_E/T}-1)^2}$$

in low temp range→approaches zero exponentially

(b) (5pts) Explain the limitation of Einstein model for the temperature dependence of C_v and how Debye could overcome this limitation briefly. Also, explain the definition of Debye temperature.

At very small temperature the experimental Cv decreases by T^3, Einstein theory predicts, instead, and exponential reduction.

<Quantum mechanical- Einstein model>

assumption: independent oscillator (one frequency) allowed energies of a single oscillator

one adjustable parameter: θ_E $(or\ w_E)$

<Quantum mechanical- Debye model>

Assumption : collective lattice modes (oscillate interdependently) $0 < \omega < \omega_D$

continuous medium $v_s = w/k$: constant

upper limit (Debye frequency, w_D) – total number of modes included are equal to the number of degrees of freedom for the entire solid ($3N_A$)

Debye temperature

$$\theta_D = \frac{\hbar\omega_D}{k} = \frac{h\nu_D}{k}$$