Midterm Exam.

1. Consider an elevator that starts in the basement and travels upward. Let N_i denote the number of people that get in the elevator at floor i, Assume that N_i's are independent and that N_i is Poisson with mean λ_i. Each person entering at i will, independent of everything else, get off at j with probability P_{ij}, $\Sigma_{j>i} P_{ij} = 1$. Let O_j be the number of people getting off the elevator at floor j.

   (a) Compute $E[O_j]$ (b) What is the distribution of O_j.

2. Suppose that at every time epoch t = 1, 2, 3, … (in units of minutes), one car arrives with probability p at a single sever gas station. A service time S of a car has distribution $P(S=1) = q$, $P(S=2) = 1-q$ (in minutes). Service times are independent of each other and of the arrival process. The station has room for at most two cars including the one being served. Arrivals who find the station full do not stop. Assume that a car arrived just as a service is completed does stop.

   (a) Draw the state transition probability diagram for a stochastic process defined on the epochs "just after" t, t = 1, 2, … , such that the process is a Markov chain. (There should be 5 states)

   (b) For $p = q = 1/2$, find (i) the proportion of time the station is full, and (ii) the expected number of cars at the station (averaged over all time).

3. Consider a birth-and-death process with the following birth and death rate

   $\lambda_k = (k+2)\lambda$ for $k=0, 1, \ldots$

   $\mu_k = k\mu$ for $k=1, 2, \ldots.$

   (a) Solve for π_k . Be sure to express your answer explicitly in terms of λ, k, and μ only

   (b) Find the average number of customers in the system

4. A taxi alternates between three locations. When it reaches location 1 it is equally likely to go next to either 2 or 3. When it reaches 2 it will next go to 1 with probability 1/3 and to 3 with probability 2/3. From 3 it always goes to 1. The moving time from location i to location j is t_{ij} according to exponential distribution. And, $t_{12}= t_{21=}20$, $t_{13}= t_{31=}30$, $t_{23}= t_{23=}30$.

   (a) What is the (limiting) probability that the taxi's most recent stop was at location i, i=1,2,3 ?

   (b) What is the (limiting) probability that taxi is heading for location 2.