## Final Exam

1. Consider an M/G/1 system in which a departing job immediately joins the front of queue with probability p or departs the system with probability (1-p). The queuing discipline is FCFS, and the service time for a returning job is independent of its previous service times. Let $\mathrm{B}^{*}(\mathrm{~s})$ be the Laplace transform for the service time and let $\mathrm{B}_{\mathrm{T}}{ }^{(s)}$ ) be the Laplace transform for the total service time of a job.
(a) Show that $\mathrm{B}_{\mathrm{T}}^{*}(\mathrm{~s})=(1-\mathrm{p}) \mathrm{B}^{*}(\mathrm{~s}) /\left\{1-\mathrm{p} \mathrm{B}^{*}(\mathrm{~s})\right\}$
(b) Find the first moment and second moment of the total service time, using $p$ and the first moment and second moment of the service time.
(c) Find the average number of jobs in the system
2. A machine shop has three machines, A, B, and C. The numbers of servers in the machine A, B, and C are one, two, and three, respectively. Service time of a server in the machine A is exponential at rate $\mu_{\mathrm{a}}$, and the service time of each server in machine B (machine C) is exponentially distributed with rate $\mu_{\mathrm{b}}\left(\mu_{\mathrm{c}}\right)$. The shop gets four types of jobs, numbered 1 through 4, where each type requires service on machines in a particular sequence; 1: ABCA, 2: CAB, 3: ACBC, 4: BCAB. The arrival process of type $i$ jobs is Poisson at rate $\lambda_{i}$.
(a) Under what conditions is this system stable?
(b) When stable, what is the joint stationary distribution of the number of jobs at each machine?
3. The jobs arrive at the queuing system with a single server according to Poisson process with rate $\lambda$. The service time of a job in the server is determined according to the exponential distribution of rate u 1 with probability p 1 , rate u 2 with probability p 2 , and rate u 3 with probability p3 (p1+p2+p3=1). Calculate the mean number of jobs in the system and the mean sojourn time in the system

4. Consider a three-station series queuing system (single exponential server at each station: the rates of station 1 , 2 , and 3 are $\mu 1, \mu 2$, and $\mu 3$, respectively) with Poisson input (rate $\lambda$ ). There are no capacity limit on the queue in the front of the first two stations (i.e., stations 1 and 2 ), but at the third there is a limit of K allowed (including service). If K customers are in the third system, then any subsequent arrivals are shunted out of the system. Find the expected number of customers in the system and the expected time spent in the system by a customer who completes all three stages of service.
5. Consider the optimization problem
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minimize \(\quad x^{2}+1\)
subject to \(\quad(x-2)(x-4) \leq 0\)
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(a) Analysis of primal problem: Give the feasible set, the optimal value, and the optimal solution
(b) Lagrangian and dual function: Plot the objective $\left(x^{2}+1\right)$ versus $x$. On the sam plot, show the feasible set, the optimal point and value, and plot Lagrangian $L(x, \lambda)$ versus $x$ for a few positive values of . Verify the lower property, i.e., $p^{*} \geq \inf _{x} L(x, \lambda)$ for $\lambda \geq 0$. Derive and sketch the Lagrangian dual function $g(\lambda)$.
(c) Lagrangian dual problem: State the dual problem, and verify that it is a concave maximization problem. Find the dual optimal value and the dual optimal solution $\lambda^{*}$. Does strong duality hold?
6. Consider the following extensive form game where the player 1 and the player 2 take their actions sequentially.

(a) Represent the strategic form
(b) Find Nash equilibria.

