SEOUL NATIONAL UNIVERSITY
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

SYSTEM ANALYSIS	**Spring 2015**
Final Exam SOLUTION	**Date: June 16, 2015 (Tu)**
Closed book, closed note	**9:30~11:30**

Student ID #: ____________________ Name: __________________

1.(20points) Describe followings:

(1) Mathematical model

Problem No (points)	Points
1(20)	
2(30)	
3(15)	
4(10)	
5(10)	
6(7)	
7(10)	
Total(102)	

(2) control systems

(3) Frequency response and bode plots

(4) Transfer Function and state equation

[2] (30 points) Electric motor system

Consider an elevator system shown in the figure 2-1. When the mass of the elevator cage is equal to that of the counter weight, a schematic diagram of the elevator system can be represented as figur 2-2. Assume that the inductance of the motor is negligible.

R_a : armature resistance, Ω
i_a : armature current, A
i_f : field current, A
e_a : applied armature voltage, V
e_b : back emf, V
K_T : Torque constant, N-m/A
K_b : *back emf constant*, V/(rad/sec)
θ_1 : angular displacement of the motor shaft, rad
θ_2 : angular displacement of the *load, i.e., elevator cage*, rad
T : torque developed by the motor, N-m
J_1 : equivalent moment of inertia of the motor to the motor shaft, kg-m^2
J_2 : equivalent moment of inertia of load, kg-m^2
b_1, b_2 : equivalent viscous-friction coefficients of the motor and load referred to the motor shaft, respectively, N-m/rad/s

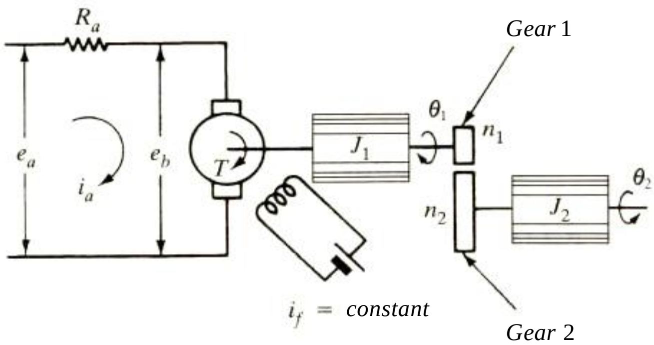


Fig. 2-2

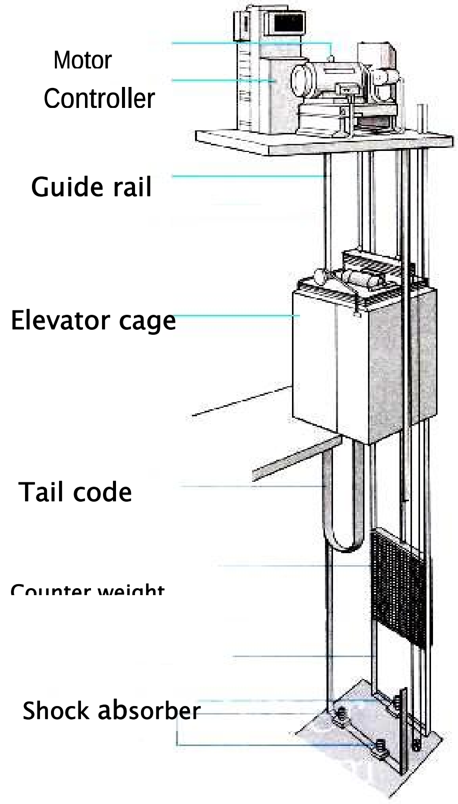


Fig. 2-1

(1) (5 points) The equation of the motion with the the motor angular position, $\theta_1(t)$, can be represented as

$$J_{eq} \frac{d^2}{dt^2}\theta_1(t) + b_{eq}\frac{d}{dt}\theta_1(t) = \mathrm{T}$$

Compute the equivalent moment of inertia, J_{eq}, and viscous damping coefficient, b_{eq}.

(2) (5 points) The position of the elevator cage is defined as

$y(t) = R \cdot \theta_2(t)$

Obtain the transfer function, $\mathrm{G(s)} = \dfrac{Y(s)}{U(s)}$ **, from the applied armature voltage input,** $\mathrm{u}(t) = e_a(t)$ **,to the position of the elevator cage,** $\mathrm{y}(t)$ **.**

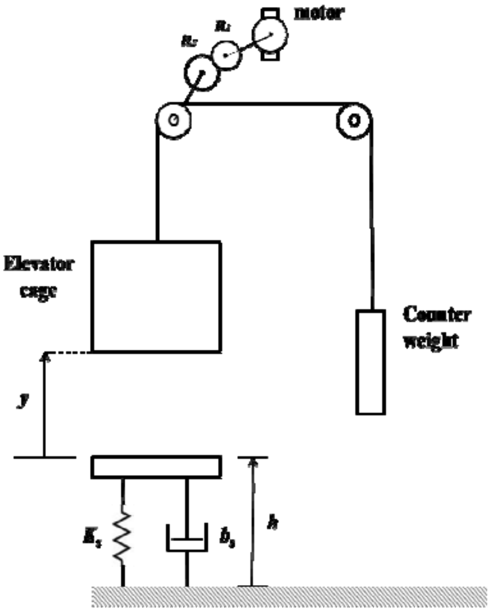


Fig. 2-3

(3) (5 points) Assume that the input $u(t) = e_a(t)$ shown in Figure 2-4 is used. Sketch the acceleration and speed profiles of the elevator cage. Note that it is not necessary to obtain the complete solution.

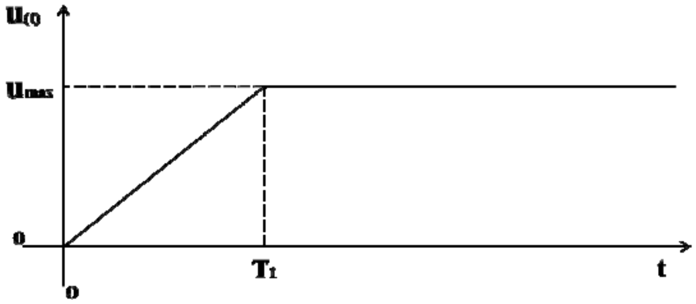


Fig. 2-4

What is the steady state speed of the elevator cage?

Note:

$$A = b_{eq} + \frac{K_T K_b}{R_a}$$

$$B = \frac{K_T}{R_a}\left(\frac{r_1}{r_2}\right)\frac{u_{max}}{T_1}$$

(4) (5 points) In this case when the input of the Figure 2-4 is applied, it is necessary to limit the acceleration of the elevator cage such that

$$a_y(t) = \frac{d^2 y(t)}{dt^2} \leq 0.3g \quad \textbf{(eq. 4-1)}$$

where g is the gravitational constant. Under what condition, for given system parameters and u_{max}, can the acceleration of the elevator cage can be limited as given in (eq. 4-1)?

(5) (10 points) Design of a shock absorber system for emergency accident: Figure 2-3. At the time $t = -T_0$, the position of the cage is y_0, i.e., $y(-T_0) = y_0$, the velocity is zero, i.e., $\dot{y}(-T_0) = 0$, and the wire connecting the elevator cage and the counter weight is cut. At the time $t = 0$, the cage hits the shock absorber system which consists of a spring and a damper. The mass of the spring-damper shock absorber system is negligible.

(5-1) (3 points) Write down the equations of motion for $-T_0 \le t < 0$ and $0 \le t$. Assume that the viscous-friction coefficient of the cage is negligible.

(5-2) (3 points) Assume that the mass of the cage including the passengers of the elevator is 1,000 kg. Design the spring and damper of the shock absorber such that:

(a) the cage does not loose contact with the shock absorber and

(b) the steady state deflection of the spring is one meter.

It is not necessary to compute the response of the cage.

(5-3) (4 points) In this case, sketch the velocity and displacement of the cage, i.e., $\dot{y}(\mathrm{t})$ and $y(t)$, for $-T_0 \le t$.

[3] (15points) Consider a system described as following state equation.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -1 \\ \frac{1}{3} & -\frac{4}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} & -\frac{8}{3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} u,$$

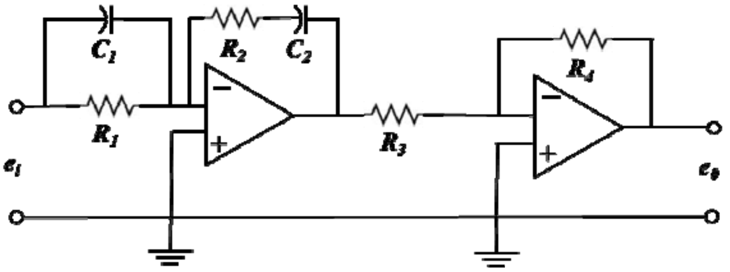
$$y = \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(1) (5 points) Transform the system into a Jordan Form.

(2) (5 points) Compute the state transition matrix of the system, i.e., $\Phi(t) = \exp(At)$ **.**

(3) (5 points) Compute the unit impulse response.

[4] (10 points) Obtain the transfer function, $G(s) = \dfrac{E_o(s)}{E_i(s)}$**, for the op-amp system shown in Figure below.**



[5] (10 points) Figure given below shows the response of a system to a step input of magnitude 100N.

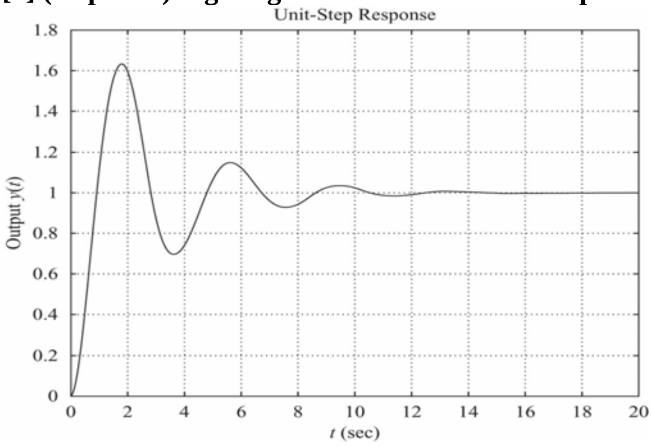


Fig. 4.1 step input response

The equation of motion is

$m\ddot{x} + c\dot{x} + kx = f(t)$

Estimate the values of ***m, c,*** **and** ***k.***

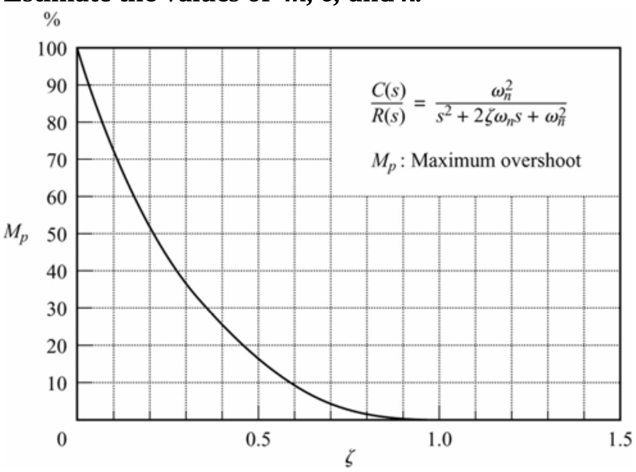
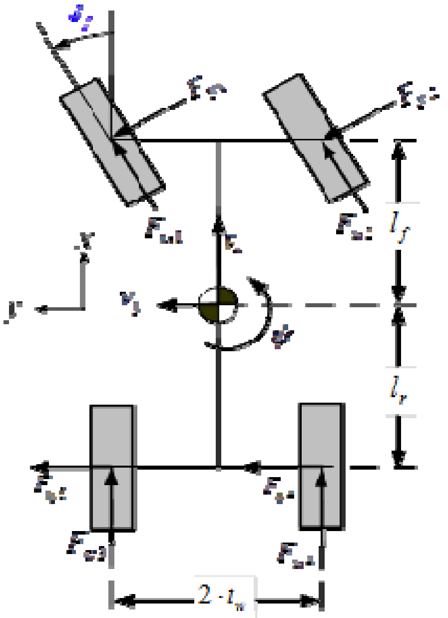


Fig. 4.2 Peak overshoot and damping ratio

[6] (7 points) Consider a vehicle steering systems shown in the figures below.



Planar vehicle dynamic model can be represented as the state equation as follows:

$$\frac{d}{dt}\begin{bmatrix}\beta\\ \gamma\end{bmatrix}=\begin{bmatrix}-\frac{2\left(C_f+C_r\right)}{mv} & -\frac{2\left(C_f l_f-C_r l_r\right)}{mv}-1\\ -\frac{2\left(C_f l_f-C_r l_r\right)}{I_z} & -\frac{2\left(C_f l_f^{\,2}+C_r l_r^{\,2}\right)}{I_z v}\end{bmatrix}\begin{bmatrix}\beta\\ \gamma\end{bmatrix}+\begin{bmatrix}\frac{2C_f}{mv}\\ \frac{2C_f l_f}{I_z}\end{bmatrix}\delta_f$$

β : vehicle body slip angle[rad]

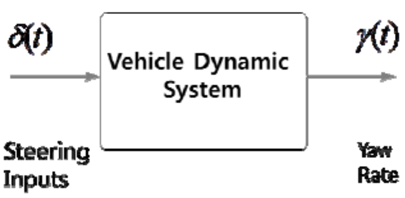
γ : yaw rate [rad / sec] = $\dot{\Psi}$

v : vehicle speed [m/sec]

C_* : cornering stiffness [N/rad]

Since the body slip angle, $\beta(t)$, is very small in normal driving situations, it can be assumed to be zero. In this case, for constant vehicle speed, the transfer function of the system can be represented as

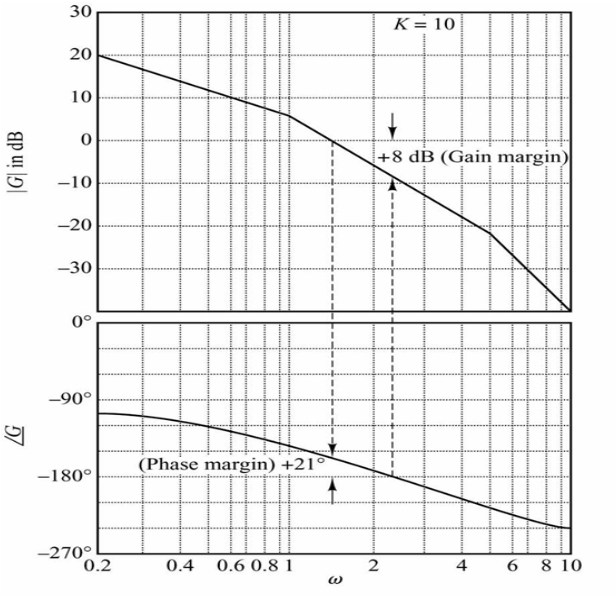
$$\gamma(s)=\frac{1}{1+\tau s}k\delta_f(s)$$



Compute the time constant, τ , and the steering gain, k . Sketch the relationship between the time constant, τ , and the vehicle speed, v , and also sketch the relationship between the steering gain, k , and the vehicle speed, v .

[7] (10 points)

(1) Frequency response of a system is shown in Figure below. The magnitude plot represents asymptotes not exact curve. Obtain the transfer function of the system.



(a)

(2) An instrument is attached to a base whose motion is to be measured. Show that the device can be used for measuring the displacement of the base. Show also that the device can be used for measuring the acceleration of the base.