Eng Math 2 - Quiz 1. Solutions

Solution 1

Inverse Matrix of $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 1 \end{pmatrix}$ by Gauss-Jordan

$\Rightarrow$
by **Gauss-Jordan** formula

$$\begin{pmatrix} 1 & 0 & 0 & \vdots & 1 & 0 & 0 \\ 2 & 1 & 0 & \vdots & 0 & 1 & 0 \\ 5 & 4 & 1 & \vdots & 0 & 0 & 1 \end{pmatrix} \rightharpoonup \begin{pmatrix} 1 & 0 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 1 & 0 & \vdots & -2 & 1 & 0 \\ 0 & 4 & 1 & \vdots & -5 & 0 & 1 \end{pmatrix} \rightharpoonup \begin{pmatrix} 1 & 0 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 1 & 0 & \vdots & -2 & 1 & 0 \\ 0 & 0 & 1 & \vdots & 3 & -4 & 1 \end{pmatrix}$$

$\therefore$ Inverse Matrix:= $\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{pmatrix}$

Check, $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Solution 2

Find **Solution** and **Unique? Or?**
given equation:

$$-2x - 4y + 7z = -6 \qquad \cdots (1)$$
$$x + 2y + 16z = 3 \qquad \cdots (2)$$

$\Rightarrow$
(1)+(2)×2 →39z=0 $\qquad$ ∴ z=0
then, from (2) $\quad$ x+2y=3

$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 - 2y \\ y \\ 0 \end{pmatrix}$

solution: $\quad$ many exist, $\quad$ $k \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$, $\quad$ $k \in \Re$

Solution 3

given equation:

$$3x_1^2 - 8x_1x_2 - 3x_2^2 = 0$$

1) Quadratic form as $q = x^TAx$

$\Rightarrow$

$q = \begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} 3 & -4 \\ -4 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

2) Eigenvalue and Eigenvector

$\Rightarrow$

$(\lambda - 3)(\lambda + 3) - 4^2 = 0$

$\therefore \lambda = \pm 5$

i) $\lambda = 5$, $\begin{pmatrix} -2 & -4 \\ -4 & -8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \longmapsto \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

ii) $\lambda = -5$, $\begin{pmatrix} 8 & -4 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \longmapsto \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

3) transfer it to principal axes

$\Rightarrow$

$y = Tx \quad T^{-1}$:Eigenmatrix

$q = x^TAx = (T^{-1}y)^TA(T^{-1}y) = y^TT^{-T}AT^{-1}y = y^T\Lambda y$

$\therefore \quad q = \begin{pmatrix} y_1 & y_2 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & -5 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

Solution 4

General Solution of the ODE,

$$\ddot{y} + w^2y = r(t)$$

$$r(t) = \sin(t) + \frac{1}{3}sin(3t) + \frac{1}{5}sin(5t) + \frac{1}{7}\sin(7t)$$

$\Rightarrow$

$$\textbf{homogeneous sol'n: } y_h = C_1\cos(wt) + C_2\sin(wt)$$

$$\textbf{partial sol'n: } y_p = \sum_{i=1}^{4}(A_i\cos(2i-1) + B_i\sin(2i-1))$$

$y = y_h + y_p = C_1\cos(wt) + C_2\sin(wt) + \sum_{i=1}^{4}(A_i\cos(2i-1) + B_i\sin(2i-1))$

$\ddot{y} = -C_1w^2\cos(wt) - C_2w^2\sin(wt) + \sum_{i=1}^{4}(-A_i(2i-1)^2\cos(2i-1) - B_i(2i-1)^2\sin(2i-1))$

into the given differential equation, $A_i = 0$ and $B_i = \frac{1}{(2i-1)(w^2-(2i-1)^2)}$, $\forall i$

$\therefore y = C_1\cos(wt) + C_2\sin(wt) + \sum_{i=1}^{4}\frac{1}{(2i-1)(w^2-(2i-1)^2)}\sin((2i-1)t)$