Eng Math 2 - Quiz 2. Solutions

Solution 1

given equation: $f(z) = ln|z| + iArg(z)$, **Analytic?**

$\Rightarrow$ let,

$z = x + iy \quad arg(z) = Arg(z) + 2n\pi$

$|z| = \sqrt{(x^2 + y^2)} = r$

then, $f(z) = ln(r) + i(\theta \pm 2npi)$

$$u = ln(r), \quad v = \theta \pm 2n\pi$$

check their **partial derivative!**

$$u_r = \frac{1}{r}, \quad v_\theta = 1 \rightarrow u_r = \frac{1}{r}v_\theta$$

$$u_\theta = 0, \quad v_r = 0 \rightarrow v_r = \frac{1}{r}u_\theta$$

$\therefore Analytic!$

Solution 2

$$\oint_C \frac{\mathrm{d}}{z^2+1}$$

$$C: \ (a)|z+i| = 1 \quad (b)|z-i| - 1$$

$\Rightarrow$

(a):

$$\oint_C \frac{\mathrm{d}}{(z-i)(z+i)} = 2\pi i \frac{1}{z-i}|_{z=-i} = 2\pi i \frac{1}{-2i} = -\pi$$

(b):

$$\oint_C \frac{\mathrm{d}}{(z+i)(z-i)} = 2\pi i \frac{1}{z+i}|_{z=i} = 2\pi i \frac{1}{2i} = \pi$$

Solution 3

integrate

$$\frac{e^{\frac{z}{2}}}{(z-a)^4}$$

around C , where C:circle $|z-2-i|=3$ (counterclockwise)

$\Rightarrow$

i) $|a-2-i|<3$

$\rightarrow$

$$\oint_C \frac{e^{\frac{z}{2}}}{(z-a)^4}\mathrm{d}z = \frac{2\pi i}{3!}(e^{\frac{z}{2}})^{(3)}|_{z=a}$$
$$= \frac{\pi i}{24}e^{\frac{a}{2}}$$

ii) $|a-2-i|>3$

$\rightarrow$

$$\oint_C \frac{e^{\frac{z}{2}}}{(z-a)^4}\mathrm{d}z = 0$$
$$(\because \ analytic)$$

Solution 4

center and **radius of convergence** of

$$\sum_{n=0}^{\infty} \frac{n^4}{2^n} z^{2n}$$

$\Rightarrow$

$$\lim_{n\to\infty}|\frac{a_{n+1}}{a_n}| = \lim_{n\to\infty}\left(\frac{(n+1)^4}{2^{n+1}}\frac{2^n}{n^4}\right) = \lim_{n\to\infty}\left(\frac{(n+1)^4}{n^4}\frac{1}{2}\right) = \frac{1}{2} = \frac{1}{R^2}$$
$$R=\sqrt{2}, \quad center : 0$$