School of Mech & Aero Eng
Seoul National University

Eng Math 2
Fall '06

MIDTERM

- Do not open exam until told to do so.
- 'How you arrived at your answer' is much more important than the answer itself. Read the following problems carefully, and make sure you show your work *step by step.*
- Please write down the answer under the given space. If you need extra space, please use a separate sheet for each problem.
- You can use the **information given in the last page** of this exam if you don't remember formulas needed to solve the problems.
- Ask questions if you don't understand what the problem says, and **GOOD LUCK !**

Student ID: ________________

Name: ________________

1	/ 20
2	/ 10
3	/ 10
4	/ 15
5	/ 15
6	/ 20
7	/ 10
Total	/ 100

1. [5+5+5+5 =20 pts]

(a) Fill in the blank space:
Let $AB = C$. Then the rows of C are combinations of the rows of ___.
So rank(C) $\leq$ rank(___). Since $B^T A^T = C^T$, rank(C) $\leq$ rank(___)

(b) True or False? Describe *briefly* "why".
v is an arbitrary vector in $\mathbb{R}^n$. Then $I - vv^T$ is diagonalizable.

(c) Prove the following:
If A is symmetric and orthogonal, eigenvalues of A are ± 1.

(d) Prove the following:
If C is the cofactor matrix of the singular matrix A, then each columns of C^T is in the nullspace of A.

2. [10 pts] As the point (x_1, x_2, x_3) in $\mathbb{R}^3$ moves on the unit sphere centered at $(0,0,0)$, what is the maximum value for
$q(x_1, x_2, x_3) = 19x_1^2 + 19x_2 + 17x_3^2 - 28x_1x_2$?

3. [10 pts] Under what condition on b_1, b_2, b_3 is the following solvable? Find all solutions when that condition holds.

$$\begin{aligned} x + 2y - 2z &= b_1 \\ 2x + 5y - 4z &= b_2 \\ 4x + 9y - 8z &= b_3 \end{aligned}$$

4. [15 pts] Show that

$$\int_0^\infty \frac{\sin w}{w} \cos xw \, dw = \begin{cases} \pi/2 & \text{if } 0 \le x < 1 \\ \pi/4 & \text{if } x = 1 \\ 0 & \text{if } x > 1\,. \end{cases}$$

5. [15 pts] Show that the solution of

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \qquad -\infty < x < \infty, t > 0$$

with $u(x,0) = f(x)$, $\frac{\partial u}{\partial t}\Big| = g(x)$ can be written as

$$u(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(F(w) \cos cwt + G(w) \frac{\sin cwt}{cw} \right) e^{-iwx} dw \ .$$

What are $F(w)$ and $G(w)$?

6. [20 pts]

The steady-state temperature $u(x, y, z)$ in the rectangular parallelepiped $0 < x < a,\ 0 < y < b,\ 0 < z < c$ satisfies

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \ .$$

If the top $(z = c)$ is kept at temperature $f(x, y)$ and the remaining sides are kept at 0 temperature, find the solution.

7. [5+5=10 pts]

(a) True or False? Describe *briefly* "why".
The functions $f(x) = x^2 - 1$ and $g(x) = x^5$ are orthogonal on $[-\pi, \pi]$.

(b) Suppose that $f(x) = x^2 + 2 \quad (0 < x < 3)$ is expanded in a Fourier series, a cosine series, and a sine series. To what value will each series converge at $x = 0$?

- Fourier series for a periodic ftn with a period 2π :

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx) \; .$$

$$\begin{aligned} a_0 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx \\ a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx \; dx \\ b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx \; dx \end{aligned}$$

- Fourier integral

$$f(x) = \frac{1}{\pi}\int_{0}^{\infty} \left(A(\omega)\cos\omega x + B(\omega)\cos\omega x\right) \; d\omega$$

$$\begin{aligned} A(\omega) &= \int_{-\infty}^{\infty} f(x)\cos\omega x \; dx \\ B(\omega) &= \int_{-\infty}^{\infty} f(x)\sin\omega x \; dx \end{aligned}$$

- Fourier transform and its inverse :

$$\mathcal{F}(f) = \hat{f}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-i\omega x} \; dx$$

$$\mathcal{F}^{-1}(\hat{f}) = f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \hat{f}(\omega)e^{i\omega x} \; d\omega$$