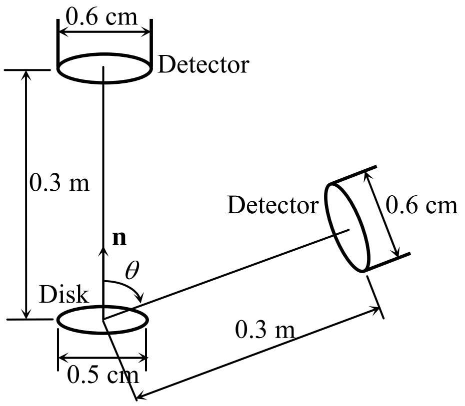
Student ID No.: Radiative Heat Transfer
Name: 2^{nd} Semester, 2006

Problem	1	2	3	4	5	Total
Score						

Mid Term Exam
(Oct. 31, 2006)

For all problems: $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4.

1. A smooth ceramic dielectric has an index of refraction n = 1.65, which is independent of wavelength. If a ceramic disk is at 675 K, how much emitted energy per unit time [W] is received by the detector when it is placed at θ = 0° and θ = 80°. Assume the index of reflection of air is unity. Use the relations from the electromagnetic theory.



$$\rho'_{\parallel}(\theta) = \left\{ \frac{(n_2/n_1)^2 \cos\theta - \left[(n_2/n_1)^2 - \sin^2\theta\right]^{1/2}}{(n_2/n_1)^2 \cos\theta + \left[(n_2/n_1)^2 - \sin^2\theta\right]^{1/2}} \right\}^2$$

$$\rho'_{\perp}(\theta) = \left\{ \frac{\left[(n_2/n_1)^2 - \sin^2\theta\right]^{1/2} - \cos\theta}{\left[(n_2/n_1)^2 - \sin^2\theta\right]^{1/2} + \cos\theta} \right\}^2$$

Solution

T = 675 K, n_1 = 1.0, n_2 = 1.65

$$Q_d = i dA_c \cos\theta d\omega = \varepsilon' i_b dA_c \cos\theta \frac{dA_d}{R^2} = \frac{1}{\pi}\varepsilon'\sigma T^4 dA_c \cos\theta \frac{dA_d}{R^2}$$

1) when θ = 0°

$$\rho'_n = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2 = \left(\frac{0.65}{2.65}\right) = 0.0602 \rightarrow \varepsilon'_n = 1 - 0.0602 = 0.9398$$

$$Q_d = \frac{1}{\pi} \times 0.9398 \times 5.67 \times 6.75^4 \times \frac{\pi \times 0.25 \times 10^{-4}}{4} \times \frac{\pi \times 0.36 \times 10^{-4}}{4 \times 0.09} = 21.7 \times 10^{-6} \text{ W}$$

2) when θ = 80°

$$\rho'_{\parallel} = \left\{ \frac{(1.65)^2 \cos 80^\circ - \left[(1.65)^2 - \sin^2 80^\circ \right]^{1/2}}{(1.65)^2 \cos 80^\circ + \left[(1.65)^2 - \sin^2 80^\circ \right]^{1/2}} \right\}^2 = 0.2244$$

$$\rho'_{\perp} = \left\{ \frac{\left[(1.65)^2 - \sin^2 80^\circ \right]^{1/2} - \cos 80^\circ}{\left[(1.65)^2 - \sin^2 80^\circ \right]^{1/2} + \cos 80^\circ} \right\}^2 = 0.5900$$

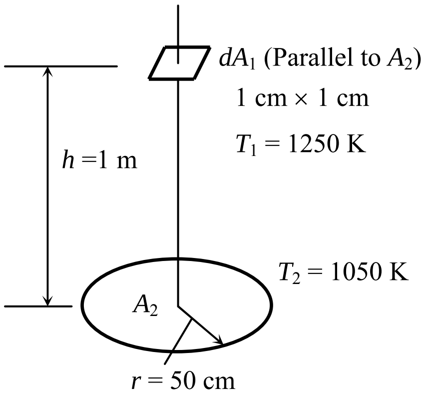
$$\rho' = \frac{0.2244 + 0.5900}{2} = 0.4072 \rightarrow \varepsilon'_n = 1 - 0.4072 = 0.5928$$

$$Q_d = \frac{1}{\pi} \times 0.5928 \times 5.67 \times 6.75^4 \times \frac{\pi \times 0.25 \times 10^{-4}}{4} \times \cos 80^\circ \times \frac{\pi \times 0.36 \times 10^{-4}}{4 \times 0.09} = 2.38 \times 10^{-6} \text{ W}$$

2. Consider the area arrangement given below.

1) Obtain the configuration factor $F_{d1\text{-}2}$ as a function of r and h using the unit sphere method.

2) What is the net energy transfer from black surface dA_1 to black surface A_2, $Q_{1\leftrightarrow 2}$ [W]?



Solution

1) $\sqrt{r^2+h^2} : r = 1 : x$

$$x = \frac{r}{\sqrt{r^2+h^2}}$$

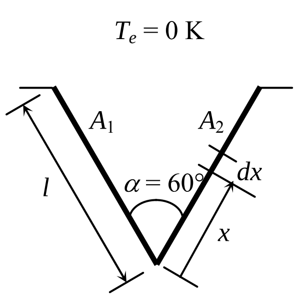
$$F_{d1-2} = \frac{A_b}{\pi} = \frac{1}{\pi}\cdot\pi\left(\frac{r}{\sqrt{r^2+h^2}}\right)^2 = \frac{r^2}{r^2+h^2}$$

2) $$Q_{1\leftrightarrow 2} = \sigma T_1^4 dA_1 F_{d1-2} - \sigma T_2^4 A_2 dF_{2-d1}$$

$$= \sigma\left(T_1^4 - T_2^4\right) dA_1 F_{d1-2}$$

$$= 5.67\times\left(12.5^4 - 10.5^4\right)\times 10^{-4}\times\frac{0.5^2}{0.5^2+1} = 1.39 \text{ W}$$

3. Consider the radiative heat loss from a long groove with V shape as shown below. One of the groove surfaces is black and the other is diffuse-gray. Both surfaces are maintained at 600 K.



A_1: black
A_2: diffuse gray, $\varepsilon_2 = 0.3$
$l = 2$ cm

1) Use simplified zone analysis and electric network analogy to find the radiative heat loss [W/m] to the environment. Specify all nodes and resistances in the network plot.
2) Use generalized zone analysis to find the radiative heat loss to the environment.

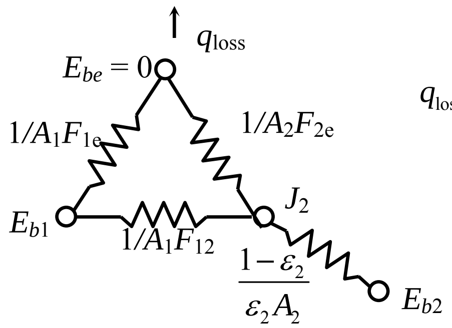
Hint: $F_{dx-l} = \frac{1}{2} + \frac{l\cos\alpha - x}{2\left(x^2 - 2xl\cos\alpha + l^2\right)^{1/2}}$

$$\int_0^l \left(1 - F_{dx-l}\right)dx = l\sin\frac{\alpha}{2}$$

$$\int_0^l F_{dx-l}\left(1 - F_{dx-l}\right)dx = \frac{(\pi-\alpha)l\sin\alpha}{8}$$

Solution

1)



$$q_{\text{loss}} = q_1 + q_2$$

$$q_1 = \frac{\sigma T_1^4}{\frac{1}{A_1F_{1e}}} + \frac{\sigma T_1^4 - J_2}{\frac{1}{A_1F_{12}}} = A_1F_{1e}\sigma T_1^4 + A_1F_{12}\left(\sigma T_1^4 - J_2\right) = \left(A_1F_{1e} + A_1F_{12}\right)\sigma T_1^4 + A_1F_{12}J_2$$

$$A_1F_{1e} = A_1F_{12} = 0.02\times 0.5 = 0.01$$

$$q_1 = 0.02\times 5.67\times 6^4 - 0.01J_2 = 147 - 0.01J_2$$

$$q_2 = \frac{\sigma T_2^4 - J_2}{\frac{1-\varepsilon_2}{A_2\varepsilon_2}} = \frac{J_2}{\frac{1}{A_2F_{2e}}} + \frac{J_2 - \sigma T_1^4}{\frac{1}{A_1F_{12}}}$$

$$\frac{A_2\varepsilon_2}{1-\varepsilon_2}\left(\sigma T_2^4 - J_2\right) = A_2F_{2e}J_2 + A_1F_{12}\left(J_2 - \sigma T_1^4\right)$$

$$\left(A_2F_{2e} + A_1F_{12} + \frac{A_2\varepsilon_2}{1-\varepsilon_2}\right)J_2 = \frac{A_2\varepsilon_2}{1-\varepsilon_2}\sigma T_2^4 + A_1F_{12}\sigma T_1^4 = \left(\frac{A_2\varepsilon_2}{1-\varepsilon_2} + A_1F_{12}\right)\sigma T_1^4$$

$$J_2 = \frac{\left(\frac{A_2\varepsilon_2}{1-\varepsilon_2} + A_1F_{12}\right)\sigma T_1^4}{2A_1F_{12} + \frac{A_2\varepsilon_2}{1-\varepsilon_2}} = \frac{\left(\frac{0.02\times 0.3}{1-0.3} + 0.02\times 0.5\right)\times 5.67\times 6^4}{2\times 0.02\times 0.5 + \frac{0.02\times 0.3}{1-0.3}} = 4776 \text{ W/m}^2$$

$$q_1 = 147 - 0.01\times 4776 = 99 \text{ W/m}$$

$$q_2 = \frac{A_2\varepsilon_2}{1-\varepsilon_2}\left(\sigma T_2^4 - J_2\right) = \frac{0.02\times 0.3}{1-0.3}\left(5.67\times 6^4 - 4778\right) = 22 \text{ W/m}$$

$$q_{\text{loss}} = q_1 + q_2 = 99 + 22 = 121 \text{ W/m}$$

2) $q_{\text{loss}} = q_{1\to e} + q_{2\to e}$

Since surface 1 is black, the radiosity $J_1 = \sigma T_1^4$ is uniform along A_1.

$$q_{1\to e} = \sigma T_1^4 A_1 F_{1e} = 5.67\times 6^4 \times 0.02\times 0.5 = 73 \text{ W/m}$$

$$q_{2\to e} = \int_{A_2} J_2(x) F_{dx-e}\, dA_2 = \int_0^l J_2(x) F_{dx-e}\, dx$$

$$J_2(x) = \varepsilon_2\sigma T_2^4 + \left(1-\varepsilon_2\right) G_2(x)$$

$$dxG_2(x) = \sigma T_1^4 l dF_{l-dx} = \sigma T_1^4 dx F_{dx-l} \to G_2(x) = \sigma T_1^4 F_{dx-l}$$

$$q_{2\to e} = \int_0^l \left[\varepsilon_2\sigma T_2^4 + \left(1-\varepsilon_2\right)\sigma T_1^4 F_{dx-l}\right] F_{dx-e}\, dx$$

$$F_{dx-e} = 1 - F_{dx-l}$$

$$q_{2\to e} = \int_0^l \left[\varepsilon_2\sigma T_2^4 + \left(1-\varepsilon_2\right)\sigma T_1^4 F_{dx-l}\right]\left(1-F_{dx-l}\right) dx$$

$$= \varepsilon_2\sigma T_2^4 \int_0^l \left(1-F_{dx-l}\right) dx + \left(1-\varepsilon_2\right)\sigma T_1^4 \int_0^l F_{dx-l}\left(1-F_{dx-l}\right) dx$$

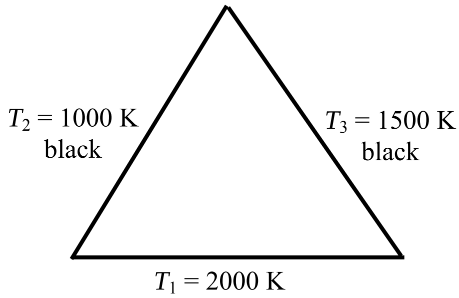
$$= \varepsilon_2\sigma T_2^4 l \sin\frac{\alpha}{2} + \left(1-\varepsilon_2\right)\sigma T_1^4 \frac{\left(\pi-\alpha\right) l \sin\alpha}{8}$$

$$= \sigma T_2^4 l\left[\varepsilon_2 \sin\frac{\alpha}{2} + \left(1-\varepsilon_2\right)\frac{\left(\pi-\alpha\right)\sin\alpha}{8}\right]$$

$$= 5.67\times 6^4 \times 0.02\times\left[0.3\times 0.5 + 0.7\times\frac{2\pi}{3}\times\frac{\sqrt{3}}{2}\times\frac{1}{8}\right] = 45 \text{ W/m}$$

$$q_{\text{loss}} = q_{1\to e} + q_{2\to e} = 75 + 45 = 120 \text{ W/m}$$

4. The equilateral triangular cross section shown below has two black sides, A_2 and A_3, and temperatures are T_2 = 1000 K and T_3 = 1500 K, respectively. The surface 1 is a diffuse one at T_1 = 2000 K but has a spectral emissivity $\varepsilon_{\lambda 1} = 0.4$ for $0 \le \lambda < 2$ μm and $\varepsilon_{\lambda 1} = 0.8$ for $\lambda \ge 2$ μm.



1) Obtain q_1 [W/m^2].
2) What is the total emissivity of surface 1, ε_1? Evaluate q_1 using the total emissivity and compare your result with that of 1).
3) If the temperature of surface 1 is higher than 2000 K, does the difference between the results of 1) and 2) become larger or samller? Explain why.

Hint: $F_{0\text{-}2000} = 0.06673$, $F_{0\text{-}3000} = 0.27323$, $F_{0\text{-}4000} = 0.48087$

Solution

1) $q_{\lambda 1} = \varepsilon_{\lambda 1} e_{\lambda b1} - \varepsilon_{\lambda 1} G_{\lambda 1} = \varepsilon_{\lambda 1} e_{\lambda b1} - \varepsilon_{\lambda 1}\left(e_{\lambda b2} F_{12} + e_{\lambda b3} F_{13}\right)$

$F_{12} = F_{13} = 0.5$

$q_{\lambda 1} = \varepsilon_{\lambda 1} e_{\lambda b1} - 0.5\varepsilon_{\lambda 1}\left(e_{\lambda b2} + e_{\lambda b3}\right)$

$$q_1 = \int_0^\infty q_{\lambda 1} d\lambda = 0.4\int_0^2 e_{\lambda b1} d\lambda - 0.2\int_0^2 \left(e_{\lambda b2} + e_{\lambda b3}\right) d\lambda + 0.8\int_2^\infty e_{\lambda b1} d\lambda - 0.4\int_2^\infty \left(e_{\lambda b2} + e_{\lambda b3}\right) d\lambda$$

$$= 0.4\sigma T_1^4 F_{0-4000} - 0.2\left(\sigma T_2^4 F_{0-2000} + \sigma T_3^4 F_{0-3000}\right)$$

$$+0.8\sigma T_1^4\left(1 - F_{0-4000}\right) - 0.4\left[\sigma T_2^4\left(1 - F_{0-2000}\right) + \sigma T_3^4\left(1 - F_{0-3000}\right)\right]$$

$$= \sigma T_1^4\left[0.4 \times 0.48087 + 0.8 \times \left(1 - 0.48087\right)\right]$$

$$-\sigma T_2^4\left[0.2 \times 0.06673 + 0.4 \times \left(1 - 0.06673\right)\right]$$

$$-\sigma T_3^4\left[0.2 \times 0.27323 + 0.4 \times \left(1 - 0.27323\right)\right]$$

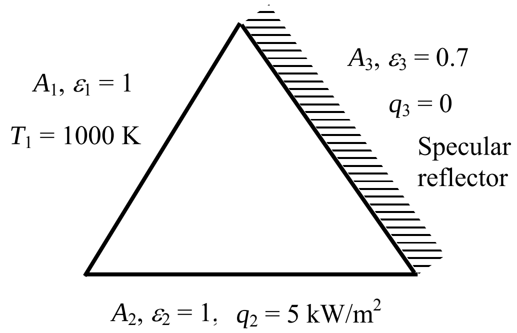
$$= 5.67 \times \left[0.607652 \times (20)^4 - 0.386654 \times (10)^4 - 0.345354 \times (15)^4\right]$$

$$= 430{,}207 \text{ W/m}^2$$

2) $\varepsilon_1 = \dfrac{\int_0^\infty \varepsilon_{\lambda 1} e_{\lambda b1} d\lambda}{\sigma T_1^4} = 0.4F_{0-4000} + 0.8\left(1 - F_{0-4000}\right) = 0.61$

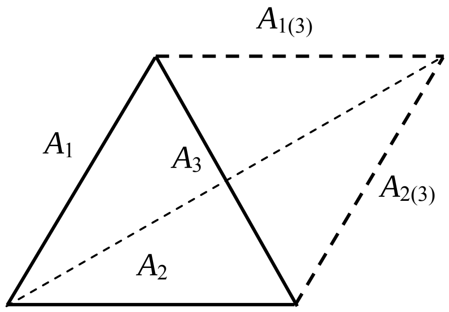
$$q_1 = \varepsilon_1 \sigma T_1^4 - \varepsilon_1\left(\sigma T_2^4 F_{12} + \sigma T_2^4 F_{13}\right) = 0.61 \times 5.67 \times \left[20^4 - 0.5 \times \left(10^4 + 15^4\right)\right] = 449{,}550 \text{ W/m}^2$$

5. An equilateral triangular enclosure has sides that extend in the normal direction infinitely far into and out of the plane of the cross section shown below.



1) Find all necessary exchange factor E needed to solve the problem.
2) Find the temperatures of surfaces 2 and 3.

Solution



$q_2 = \varepsilon_2 \left(\sigma T_2^4 - G_2\right)$, $G_2 = J_1 E_{21} + J_2 E_{22} + \varepsilon_3 \sigma T_3^4 E_{23}$

$J_1 = \sigma T_1^4$, $J_2 = \sigma T_2^4$

$$E_{21} = F_{21} + \rho_3^S F_{2(3)-1} = 0.5 + 0.3 \times \frac{\sqrt{3}-1}{2} = 0.610$$

$$E_{22} = \rho_3^S F_{2(3)-2} = 0.3 \times \frac{2-\sqrt{3}}{2} = 0.040$$

$E_{23} = F_{23} = 0.5$

$q_2 = 5000 = \varepsilon_2 \left(\sigma T_2^4 - G_2\right) = \sigma\left(0.96 T_2^4 - 0.61 T_1^4 - 0.35 T_3^4\right)$

$0.96 T_2^4 - 0.35 T_3^4 = 5000/\sigma + 0.61 T_1^4 = 0.698 \times 10^{12}$ (1)

$q_3 = \varepsilon_3 \left(\sigma T_3^4 - G_3\right) = 0 \rightarrow \sigma T_3^4 = G_3$

$G_3 = J_1 E_{31} + J_2 E_{32} + \varepsilon_3 \sigma T_3^4 E_{33}$

$J_1 = \sigma T_1^4$, $J_2 = \sigma T_2^4$

$E_{31} = F_{31} = 0.5$, $E_{32} = F_{32} = 0.5$, $E_{33} = 0$

$\sigma T_3^4 = 0.5 \sigma T_1^4 + 0.5 \sigma T_2^4$ or $0.5 T_2^4 - T_3^4 = -0.5 T_1^4 = -0.5 \times 10^{12}$ (2)

$$T_2^4 = \frac{\begin{vmatrix} 0.698\times10^{12} & -0.35 \\ -0.5\times10^{12} & -1 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.35 \\ 0.5 & -1 \end{vmatrix}} = \frac{0.873\times10^{12}}{0.785} \rightarrow T_2 = 1027\text{ K}$$

$$T_3^4 = 0.5\times\frac{0.873\times10^{12}}{0.785} + 0.5\times10^{12} \rightarrow T_3 = 1014\text{ K}$$