Student ID No.: Radiative Heat Transfer
Name: 2^{nd} Semester, 2006

Problem	1	2	3	4	5	Total
Score						

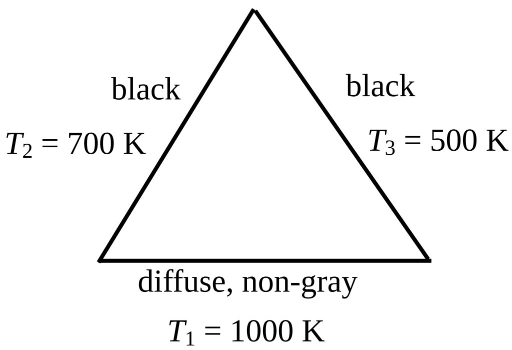
Final Exam

(December 14, 2006)

For all problems: $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4, $F_{0\text{-}2000} = 0.06673$, $F_{0\text{-}1400} = 0.00779$, $F_{0\text{-}1000} = 0.000321$

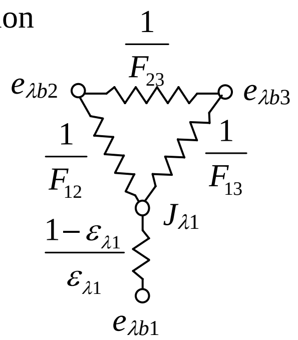
1.(20) Consider a two-dimensional equilateral triangular geometry given below.
(a)(12) Find the radiative heat flux from surface 1, q_1'' [W/m^2].
(b)(3) What is the total emissivity of surface 1, ε_1?
(c)(5) If you assume that surface 1 is gray with the emissivity you find in problem (b), what is q_1'' ?

The spectral emissivity of surface 1 is: $\varepsilon_{\lambda 1} = \begin{cases} 0.8 & 0 \le \lambda < 2\mu\text{m} \\ 0.5 & \lambda \ge 2\mu\text{m} \end{cases}$



Solution

a)



$$q_{\lambda 1}'' = \frac{e_{\lambda b1} - J_{\lambda 1}}{\frac{1-\varepsilon_{\lambda 1}}{\varepsilon_{\lambda 1}}} = \frac{J_{\lambda 1} - e_{\lambda b2}}{\frac{1}{F_{12}}} + \frac{J_{\lambda 1} - e_{\lambda b3}}{\frac{1}{F_{13}}}$$

Since $F_{12} = F_{13}$ (= 0.5)

$$e_{\lambda b1} - J_{\lambda 1} = \left(\frac{1-\varepsilon_{\lambda 1}}{\varepsilon_{\lambda 1}}\right) F_{12}\left(2J_{\lambda 1} - e_{\lambda b2} - e_{\lambda b3}\right)$$

$$\left[2\left(\frac{1-\varepsilon_{\lambda 1}}{\varepsilon_{\lambda 1}}\right)F_{12} + 1\right] J_{\lambda 1} = e_{\lambda b1} + \left(\frac{1-\varepsilon_{\lambda 1}}{\varepsilon_{\lambda 1}}\right) F_{12}\left(e_{\lambda b2} + e_{\lambda b3}\right)$$

$$J_{\lambda 1} = \frac{e_{\lambda b1} + \left(\frac{1-\varepsilon_{\lambda 1}}{\varepsilon_{\lambda 1}}\right) F_{12}\left(e_{\lambda b2} + e_{\lambda b3}\right)}{2\left(\frac{1-\varepsilon_{\lambda 1}}{\varepsilon_{\lambda 1}}\right) F_{12} + 1}$$

$$q''_{\lambda 1} = \frac{e_{\lambda b1} - J_{\lambda 1}}{\frac{1-\varepsilon_{\lambda 1}}{\varepsilon_{\lambda 1}}} = \frac{\varepsilon_{\lambda 1}}{1-\varepsilon_{\lambda 1}}\left[e_{\lambda b1} - \frac{e_{\lambda b1} + \left(\frac{1-\varepsilon_{\lambda 1}}{\varepsilon_{\lambda 1}}\right) F_{12}\left(e_{\lambda b2} + e_{\lambda b3}\right)}{2\left(\frac{1-\varepsilon_{\lambda 1}}{\varepsilon_{\lambda 1}}\right) F_{12} + 1} \right]$$

$$= \frac{\varepsilon_{\lambda 1}}{1-\varepsilon_{\lambda 1}}\left[\left\{ 1 - \frac{1}{2\left(\frac{1-\varepsilon_{\lambda 1}}{\varepsilon_{\lambda 1}}\right) F_{12} + 1} \right\} e_{\lambda b1} - \frac{\left(\frac{1-\varepsilon_{\lambda 1}}{\varepsilon_{\lambda 1}}\right) F_{12}}{2\left(\frac{1-\varepsilon_{\lambda 1}}{\varepsilon_{\lambda 1}}\right) F_{12} + 1}\left(e_{\lambda b2} + e_{\lambda b3}\right) \right]$$

$$q''_1 = \int_0^\infty q''_{\lambda 1} d\lambda = \int_0^2 q''_{\lambda 1} d\lambda + \int_2^\infty q''_{\lambda 1} d\lambda$$

$$= \frac{0.8}{1-0.8}\left[\left\{ 1 - \frac{1}{2\left(\frac{1-0.8}{0.8}\right)\times 0.5 + 1} \right\} \int_0^2 e_{\lambda b1} d\lambda - \frac{\left(\frac{1-0.8}{0.8}\right)\times 0.5}{2\left(\frac{1-0.8}{0.8}\right)\times 0.5 + 1}\left(\int_0^2 e_{\lambda b2} d\lambda + \int_0^2 e_{\lambda b3} d\lambda \right) \right]$$

$$+ \frac{0.5}{1-0.5}\left[\left\{ 1 - \frac{1}{2\left(\frac{1-0.5}{0.5}\right)\times 0.5 + 1} \right\} \int_2^\infty e_{\lambda b1} d\lambda - \frac{\left(\frac{1-0.5}{0.5}\right)\times 0.5}{2\left(\frac{1-0.5}{0.5}\right)\times 0.5 + 1}\left(\int_2^\infty e_{\lambda b2} d\lambda + \int_2^\infty e_{\lambda b3} d\lambda \right) \right]$$

$$= 0.8\sigma T_1^4 F_{0-2000} - 0.4\left(\sigma T_2^4 F_{0-1400} + \sigma T_3^4 F_{0-1000}\right)$$
$$+0.5\sigma T_1^4\left(1 - F_{0-2000}\right) - 0.25\left[\sigma T_2^4\left(1 - F_{0-1400}\right) + \sigma T_3^4\left(1 - F_{0-1000}\right)\right]$$

$$= 5.67\left[\begin{array}{l} 0.8\times 10^4 \times 0.06673 - 0.4\left(7^4 \times 0.00779 + 5^4 \times 0.000321\right) \\ +0.5\times 10^4 \times 0.93327 - 0.25\left(7^4 \times 0.99221 + 5^4 \times 0.999679\right) \end{array} \right]$$

$$= 25{,}180 \text{ W/m}^2$$

b) $\varepsilon_1 = 0.8F_{0-2000} + 0.5\left(1 - F_{0-2000}\right) = 0.8\times 0.06673 + 0.5\times 0.93327 = 0.52$

c) $q''_1 = \dfrac{e_{b1} - J_1}{\frac{1-\varepsilon_1}{\varepsilon_1}} = \dfrac{J_1 - e_{b2}}{\frac{1}{F_{12}}} + \dfrac{J_1 - e_{b3}}{\frac{1}{F_{13}}}$

$$e_{b1} - J_1 = \left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) F_{12}\left(2J_1 - e_{b2} - e_{b3}\right)$$

$$\left[2\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right)F_{12}+1\right]J_1 = e_{b1}+\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right)F_{12}\left(e_{b2}+e_{b3}\right)$$

$$J_1 = \frac{e_{b1}+\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right)F_{12}\left(e_{b2}+e_{b3}\right)}{2\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right)F_{12}+1}$$

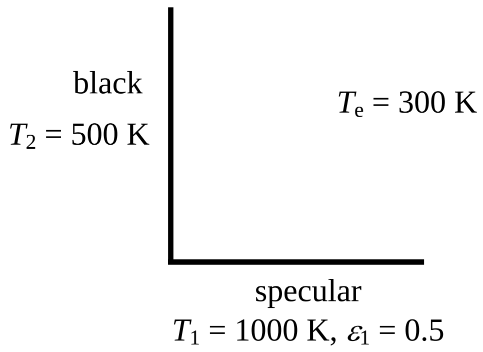
$$q_1'' = \frac{\varepsilon_1}{1-\varepsilon_1}\left(e_{b1}-J_1\right) = \frac{\varepsilon_1}{1-\varepsilon_1}\left[e_{b1}-\frac{e_{b1}+\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right)F_{12}\left(e_{b2}+e_{b3}\right)}{2\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right)F_{12}+1}\right]$$

$$= \frac{\varepsilon_1}{1-\varepsilon_1}\left[\left(1-\frac{1}{2\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right)F_{12}+1}\right)\sigma T_1^4-\frac{\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right)F_{12}}{2\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right)F_{12}+1}\left(\sigma T_2^4+\sigma T_3^4\right)\right]$$

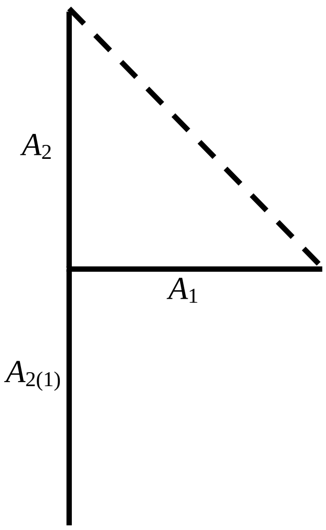
$$= \frac{0.52}{1-0.52}\left[\left(1-\frac{1}{2\left(\frac{1-0.52}{0.52}\right)\times 0.5+1}\right)\sigma T_1^4-\frac{\left(\frac{1-0.52}{0.52}\right)\times 0.5}{2\left(\frac{1-0.52}{0.52}\right)\times 0.5+1}\left(\sigma T_2^4+\sigma T_3^4\right)\right]$$

$$= \frac{0.52}{1-0.52}\times 5.67\left[0.48\times 10^4-0.24\left(7^4+5^4\right)\right] = 25,023\ \text{W/m}^2$$

2.(20) Find q_1' and q_2' [W/m] from plate 1 and plate 2 as shown below. Plate 1 is specular surface with emissivity of ε_1 = 0.5, and plate 2 is black. Both plates are 1 m long. The environmental temperature is T_e = 300 K.



Solution



$$q_1' = \varepsilon_1\left(\sigma T_1^4 - G_1\right),\ G_1 = \sigma T_2^4 E_{12} + \sigma T_e^4 E_{1e}$$

$$E_{12} = F_{12} = \frac{1+1-\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2}$$

$$E_{1e} = F_{1e} = \frac{1+\sqrt{2}-1}{2} = \frac{\sqrt{2}}{2}$$

$$q_1' = \varepsilon_1\left(\sigma T_1^4 - \sigma T_2^4 F_{12} - \sigma T_e^4 F_{1e}\right)$$

$$= 0.5\times 5.67\times\left(10^4 - 5^4\times\frac{2-\sqrt{2}}{2} - 3^4\times\frac{\sqrt{2}}{2}\right)$$

$$= 27,669 \text{ W/m}$$

$$q_2' = \sigma T_2^4 - G_2,\ G_2 = \varepsilon_1 \sigma T_1^4 E_{21} + \sigma T_e^4 E_{2e}$$

$$E_{21} = F_{21} = F_{12} = \frac{2-\sqrt{2}}{2},\ E_{2e} = F_{2e} + \rho_1 F_{2(1)-e}$$

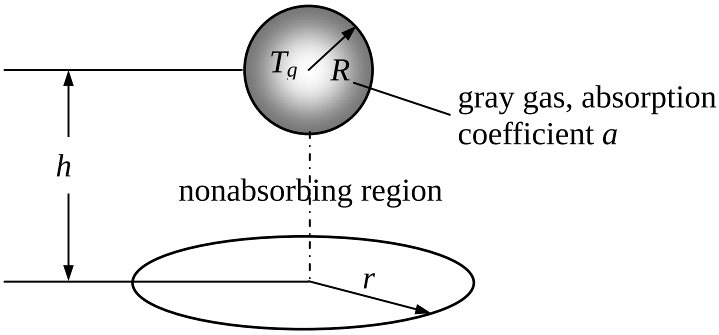
$$F_{2e} = F_{1e} = \frac{\sqrt{2}}{2},\ F_{2(1)-e} = \frac{2+1-1-\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2}$$

$$q_2' = \sigma T_2^4 - \varepsilon_1 \sigma T_1^4 F_{21} - \sigma T_e^4\left(F_{2e} + \rho_1 F_{2(1)-e}\right)$$

$$= 5.67\times\left[5^4 - 0.5\times 10^4\times\frac{2-\sqrt{2}}{2} - 3^4\times\left(\frac{\sqrt{2}}{2} + 0.5\times\frac{2-\sqrt{2}}{2}\right)\right]$$

$$= -5,152 \text{ W/m}$$

3.(20) A sphere of gray gas at uniform temperature, T_g, is situated above a surface with the region between the sphere and surface being nonabsorbing. We are looking for the amount of radiative energy incident on the circular disk from the sphere of gas.



Let's first consider the radiative energy coming out through the entire spherical boundary by emission from the spherical gas. The geometric mean transmittance $\bar{\tau}_{jk}$ from entire sphere, A_j, to its entire surface, A_k, is given by

$$\bar{\tau}_{jk} = \frac{2}{(2aR)^2}\left[1-(2aR+1)\exp(-2aR)\right]$$

The irradiation on A_k from A_j and from the spherical gas volume can be expressed as follows as we discussed in the class.

$$G_{k,j-k}A_k = \left(A_j F_{jk}\bar{\tau}_{jk}\right)J_j + \left(A_j F_{jk}\bar{\alpha}_{jk}\right)e_{b,g}$$

If you assume the radiosity from the spherical boundary is negligible, it can be expressed as $G_{k,j-k}A_k = \left(A_j F_{jk}\bar{\alpha}_{jk}\right)e_{b,g}$

(a)(3) What is the value of F_{jk} in this case?

(b)(10) Find the expression for the radiative energy q_s[W] emitted from the spherical gas volume to its boundary (= $G_{k,j-k}A_k$) as a function of R, T_g, and a.

(c)(7) Now find the expression for the radiative energy q[W] incident on the circular disk from the sphere of gas. The configuration factor between the sphere and the circular disk is given by $\frac{1}{2}\left(1-\frac{h}{\sqrt{h^2+r^2}}\right)$.

Solution

(a) $F_{jk}=1$

(b) $q_s = G_{k,j-k}A_k = \left(A_j F_{jk}\bar{\alpha}_{jk}\right)e_{b,g} = 4\pi R^2\left(1-\bar{\tau}_{jk}\right)\sigma T_g^4$

$$= 4\pi R^2\sigma T_g^4\left\{1-\frac{2}{(2aR)^2}\left[1-(2aR+1)\exp(-2aR)\right]\right\}$$

(c) $q = 4\pi R^2\sigma T_g^4\left\{1-\frac{2}{(2aR)^2}\left[1-(2aR+1)\exp(-2aR)\right]\right\}\frac{1}{2}\left(1-\frac{h}{\sqrt{h^2+r^2}}\right)$

$$= 2\pi R^2\sigma T_g^4\left(1-\frac{h}{\sqrt{h^2+r^2}}\right)\left\{1-\frac{2}{(2aR)^2}\left[1-(2aR+1)\exp(-2aR)\right]\right\}$$

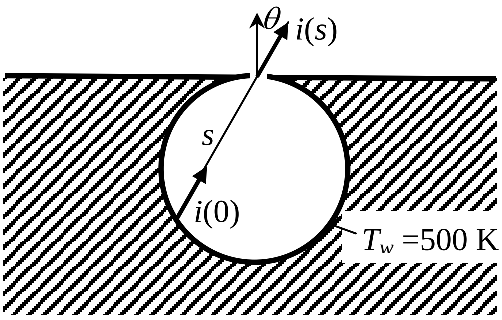
4.(20) A spherical cavity with diameter D = 20 cm is filled with a gray medium having an absorption coefficient of a = 0.1 cm^{-1}. The cavity surface is black and is at a uniform temperature of T_w = 500 K. When the medium is first placed in the cavity, the medium is cold. For this condition use the cold medium approximation to estimate the heat flux radiated out of the small opening as shown.

(a)(3) Neglecting scattering and with the cold medium approximation, write down the radiative transfer equation for $i(s)$.

(b)(5) Find the solution to the above equation for $i(s)$.

(c)(12) Estimate the heat flux radiated out of the small opening as shown.

Hint: $\int x e^{ax} dx = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right)$



Solution

(a) $\frac{di(s)}{ds} + a i(s) = 0$

(b) $\frac{d}{ds}\left[i(s)e^{as}\right] = 0,\ i(s)e^{as} - i(0) = 0,\ i(s) = i(0)e^{-as}$

$$i(0) = \frac{\sigma T_w^4}{\pi},\ i(s) = \frac{\sigma T_w^4}{\pi} e^{-as}$$

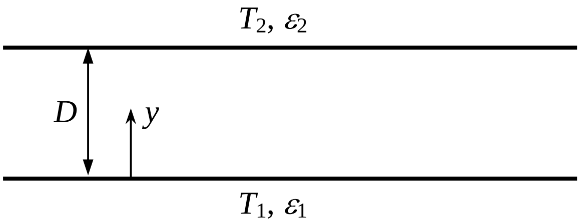
(c) $q'' = \int_{\cap} i(s)\cos\theta d\omega,\quad s = 2R\cos\theta = D\cos\theta$

$$q'' = 2\pi \frac{\sigma T_w^4}{\pi}\int_0^{\pi/2} e^{-aD\cos\theta}\cos\theta\sin\theta d\theta = 2\sigma T_w^4 \int_0^1 \mu e^{-aD\mu} d\mu$$

$$= 2\sigma T_w^4\left[-\frac{e^{-aD\mu}}{aD}\left(\mu + \frac{1}{aD}\right)\right]_0^1 = 2\sigma T_w^4\left[-\frac{e^{-aD}}{aD}\left(1 + \frac{1}{aD}\right) + \frac{1}{(aD)^2}\right]$$

$$= 2\times 5.67\times 5^4\left[-\frac{e^{-2}}{2}\left(1+\frac{1}{2}\right) + \frac{1}{4}\right] = 1052.5 \text{ W/m}^2$$

5.(20) Consider two infinite parallel plates whose temperatures and emissivities are T_1, ε_1, and T_2, ε_2. A gray midium which is optically thick is filled between the plates. The absorption coefficient of the medium is a.



For an optically thick planar medium the radiative heat flux can be approximated as

$$q'' = -\frac{4\pi}{3}\frac{di_b}{d\tau}$$

and jump boundary conditions are given by

$$\sigma\left[T_1^4 - T^4(0)\right] = \left(\frac{1}{\varepsilon_1} - \frac{1}{2}\right)q'', \qquad \sigma\left[T^4(\tau_0) - T_2^4\right] = \left(\frac{1}{\varepsilon_2} - \frac{1}{2}\right)q''$$

where $\tau = ay$ and $\tau_0 = aD$.

(a)(12) Find the expression for radiative heat flux between two plates using jump boundary conditions.

Now when D = 5 cm, T_1 = 700 K, ε_1 = 0.8, T_2 = 500 K, and ε_2 = 0.3,
(b)(5) compute the heat transferred by radiation across the gap between the plates when the gap is a vacuum,
(c)(3) and when the gap is filled with a gray medium of absorption coefficient a = 0.6 cm^{-1}.

Solution

(a) $q'' = -\dfrac{4\pi}{3}\dfrac{di_b}{d\tau} = -\dfrac{4\sigma}{3}\dfrac{dT^4}{d\tau} = \text{constant}$

$$\int_0^{\tau_0}\frac{3}{4}q''d\tau = -\int_0^{\tau_0}\sigma dT^4, \quad \frac{3}{4}q''\tau_0 = \sigma\left[T^4(0) - T^4(\tau_0)\right]$$

From jump boundary conditions,

$$\sigma T^4(0) = \sigma T_1^4 - \left(\frac{1}{\varepsilon_1} - \frac{1}{2}\right)q'', \; \sigma T^4(\tau_0) = \sigma T_2^4 + \left(\frac{1}{\varepsilon_2} - \frac{1}{2}\right)q''$$

$$\frac{3}{4}q''\tau_0 = \sigma\left(T_1^4 - T_2^4\right) - \left(\frac{1}{\varepsilon_1} - \frac{1}{2} + \frac{1}{\varepsilon_2} - \frac{1}{2}\right)q'' = \sigma\left(T_1^4 - T_2^4\right) - \left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)q''$$

$$\frac{3}{4}\tau_0 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{q''} - \left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) \quad \text{or} \quad \frac{\sigma\left(T_1^4 - T_2^4\right)}{q''} = \frac{3}{4}\tau_0 + \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1$$

Thus, $q'' = \dfrac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{3}{4}\tau_0 + \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$

(b) $q'' = \dfrac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \dfrac{5.67\left(7^4 - 5^4\right)}{\frac{1}{0.8} + \frac{1}{0.3} - 1} = 2810\ \text{W/m}^2$

(c) $q'' = \dfrac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{3}{4}\tau_0 + \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \dfrac{5.67\left(7^4 - 5^4\right)}{\frac{3}{4}\times 0.6 \times 5 + \frac{1}{0.8} + \frac{1}{0.3} - 1} = 1726\ \text{W/m}^2$