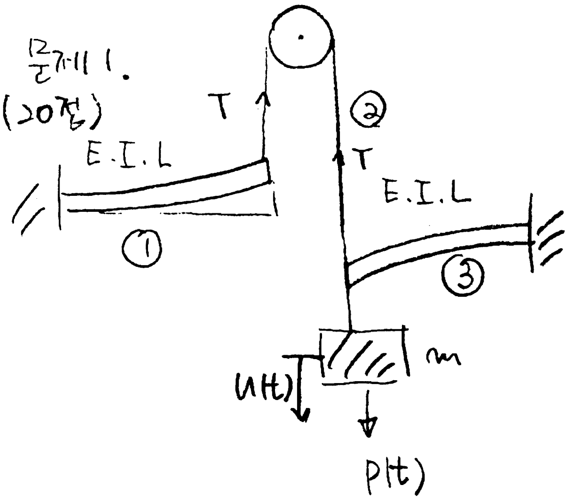
문제 1.
(20점)



① 빔의 강성

$$k_1 = \frac{3EI}{L^3}$$

②의 cable에 도입되는 장력 T 동일

③ 빔의 강성

$$k_2 = \frac{3EI}{L^3}$$

10점

두 beam은 변위가 모두 동일하게 발생

⇒ 병렬 spring

$$m\ddot{u}(t) + \frac{6EI}{L^3}u(t) = p(t)$$

17점

진동수 $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{6EI}{mL^3}}$

20점

문제 2.
(20점)

$$TR = \left\{ \frac{1 + [2\xi(\omega/\omega_n)]^2}{[1-(\omega/\omega_n)^2]^2 + (2\xi\ \omega/\omega_n)^2} \right\}^{1/2}$$

3점

Case 1. $\omega = \omega_n$에서 $TR \leq 2$

$$\omega/\omega_n = 1$$

$$TR = \left\{ \frac{1 + (2\xi)^2}{(2\xi)^2} \right\}^{1/2} \leq 2$$

$$1 + (2\xi)^2 \leq 4(2\xi)^2$$

$$12\xi^2 \geq 1 \qquad \xi \geq 0.2887$$

12점

case 2. $\omega = 2\pi f = 10\pi$ $\quad \omega/\omega_n = k$라두면 $\quad TR \le 0.2$

$$TR = \left\{ \frac{1+(2\times 0.2887\times k)^2}{(1-k^2)^2+(2\times 0.2887\, k)^2} \right\}^{1/2} \le 0.2$$

14점

$$1+(0.5774k)^2 \le 0.04\left[(1-k^2)^2+(0.5774\,k)^2\right]$$

$$1+(0.5774)^2k^2 \le 0.04\left(1-2k^2+k^4+(0.5774)^2k^2\right)$$

$$1+(0.5774)^2k^2 \le 0.04-0.08k^2+\underline{0.04k^4}+0.04\times(0.5774)^2k^2$$

$$0.04k^4+\left(0.04\times(0.5774)^2-0.08-(0.5774)^2\right)k^2+(0.04-1)\ge 0$$

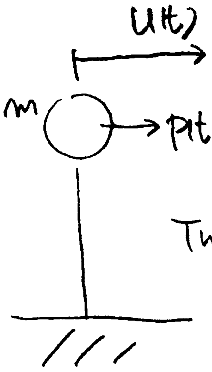
$$k^2 \ge 12.0$$

$$\left(\frac{\omega}{\omega_n}\right)^2 \ge 12.0 \qquad f_n \le 1.44\ Hz$$

$$\omega/\omega_n \ge \sqrt{12.0} \qquad \omega_n \le \frac{10\pi}{\sqrt{12.0}} = 9.07\ rad/s$$

20점

문제 3.
(35점)



$T_n = 2sec$

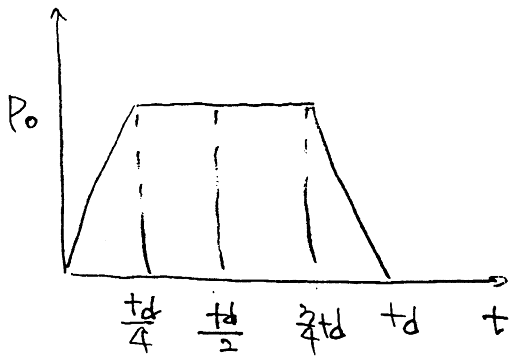
$$\omega_n = \sqrt{\frac{k}{m}} = \frac{2\pi}{T_n} \qquad T_n = 2\pi\sqrt{\frac{m}{k}}$$

$$\underline{\omega_n = \pi}$$

(20점)
가. 지붕의 수평변위 u(t)에 대한 지배운동 방정식을 유도

$$\underline{m\ddot{u}(t)+ku(t)=p(t)}$$

3점



$$m\ddot{u} + ku = p(t) = \begin{cases} P_o \dfrac{t}{\left(\frac{t_d}{4}\right)} = P_o \dfrac{4t}{t_d} & t \le \dfrac{t_d}{4} \\ P_o & \dfrac{t_d}{4} \le t \le \dfrac{3}{4}t_d \\ 4P_o\left(1 - \dfrac{t}{t_d}\right) & \dfrac{3}{4}t_d \le t \le t_d \end{cases}$$

$$u(t) = \frac{1}{m\omega_n}\int_0^{t_d} p(\tau)\sin[\omega_n(t-\tau)]\,d\tau$$

$$= \frac{1}{m\omega_n}\int_0^{\frac{t_d}{4}} P_o \frac{4\tau}{t_d}\sin\omega_n(t-\tau)\,d\tau$$

$$+ \frac{1}{m\omega_n}\int_{\frac{t_d}{4}}^{\frac{3}{4}t_d} P_o \sin\omega_n(t-\tau)\,d\tau$$

$$+ \frac{1}{m\omega_n}\int_{\frac{3}{4}t_d}^{t_d} 4P_o\left(1 - \frac{\tau}{t_d}\right)\sin\omega_n(t-\tau)\,d\tau$$

8강

$$\int \tau \sin a\tau\, d\tau = \frac{1}{a^2}\sin a\tau - \frac{\tau}{a}\cos a\tau$$ 이용

$$u(t) = \frac{1}{m\omega_n}\frac{4P_o}{t_d}\left[\frac{1}{(-\omega_n)^2}\sin\omega_n(t-\tau) + \frac{\tau}{\omega_n}\cos\omega_n(t-\tau)\right]_0^{\frac{t_d}{4}}$$

$$+\frac{1}{m\omega_n}P_o\left[+\frac{1}{\omega_n}\cos\omega_n(t-\tau)\right]_{\frac{t_d}{4}}^{\frac{3}{4}t_d}$$

$$+\frac{1}{m\omega_n}4P_o\left[+\frac{1}{\omega_n}\cos\omega_n(t-\tau)\right]_{\frac{3}{4}t_d}^{t_d}$$

$$-\frac{1}{m\omega_n}\frac{4P_o}{t_d}\left[\frac{1}{\omega_n^2}\sin\omega_n(t-\tau) + \frac{\tau}{\omega_n}\cos\omega_n(t-\tau)\right]_{\frac{3}{4}t_d}^{t_d}$$

$$= \frac{1}{m\omega_n}\frac{4P_o}{t_d}\left[\frac{1}{\omega_n^2}\left(\sin\underset{\pi}{\overset{}{\omega_n}}(t-\frac{t_d}{4}) - \sin\underset{\pi}{\omega_n t}\right) + \frac{t_d}{4\omega_n}\cos\omega_n(t-\frac{t_d}{4})\right]$$

$$+\frac{1}{m\omega_n}P_o\left[+\frac{1}{\omega_n}\left(\cos\omega_n(t-\frac{3}{4}t_d) - \cos\omega_n(t-\frac{t_d}{4})\right)\right]$$

$$+\frac{4P_o}{m\omega_n^2}\left[\cos\omega_n(t-t_d) - \cos\omega_n(t-\frac{3}{4}t_d)\right]$$

$$-\frac{1}{m\omega_n}\frac{4P_o}{t_d}\left[\frac{1}{\omega_n^2}\left(\sin\omega_n(t-t_d) - \sin\omega_n(t-\frac{3}{4}t_d)\right)\right.$$

$$\left.+\frac{t_d}{\omega_n}\cos\omega_n(t-t_d) - \frac{3t_d}{4\omega_n}\cos\omega_n(t-\frac{3}{4}t_d)\right]$$

$$= \frac{1}{m\omega_n^3}\frac{4P_o}{t_d}\sin\omega_n\left(t-\frac{t_d}{4}\right) - \frac{1}{m\omega_n^3}\frac{4P_o}{t_d}\sin\omega_n t$$

$$+ \cancel{\frac{P_o}{m\omega_n^2}\cos\omega_n\left(t-\frac{t_d}{4}\right)} + \cancel{\frac{P_o}{m\omega_n^2}\cos\omega_n\left(t-\frac{3}{4}t_d\right)} - \cancel{\frac{P_o}{m\omega_n^2}\cos\omega_n\left(t-\frac{t_d}{4}\right)}$$

$$+ \cancel{\frac{4P_o}{m\omega_n^2}\cos\omega_n(t-t_d)} - \cancel{\frac{4P_o}{m\omega_n^2}\cos\omega_n\left(t-\frac{3}{4}t_d\right)}$$

$$- \frac{1}{m\omega_n^3}\frac{4P_o}{t_d}\sin\omega_n(t-t_d) + \frac{1}{m\omega_n^3}\cdot\frac{4P_o}{t_d}\sin\omega_n\left(t-\frac{3}{4}t_d\right)$$

$$- \cancel{\frac{4P_o}{m\omega_n^2}\cos\omega_n(t-t_d)} + \cancel{\frac{3P_o}{m\omega_n^2}\cos\omega_n\left(t-\frac{3}{4}t_d\right)}$$

$$u(t) = \frac{1}{m\omega_n^3}\cdot\frac{4P_o}{t_d}\sin\omega_n\left(t-\frac{t_d}{4}\right) - \frac{1}{m\omega_n^3}\frac{4P_o}{t_d}\sin\omega_n t$$

$$- \frac{1}{m\omega_n^3}\frac{4P_o}{t_d}\sin\omega_n(t-t_d) + \frac{1}{m\omega_n^3}\frac{4P_o}{t_d}\sin\omega_n\left(t-\frac{3}{4}t_d\right)$$

12점

$$\dot{u}(t) = \frac{1}{m\omega_n^3}\frac{4P_o}{t_d}\cdot\omega_n\cos\omega_n\left(t-\frac{t_d}{4}\right) - \frac{1}{m\omega_n^3}\frac{4P_o}{t_d}\omega_n\cos\omega_n t$$

$$- \frac{1}{m\omega_n^3}\frac{4P}{t_d}\omega_n\cos\omega_n(t-t_d) + \frac{1}{m\omega_n^3}\frac{4P_o}{t_d}\omega_n\cos\omega_n\left(t-\frac{3}{4}t_d\right)$$

$$= \frac{1}{m\omega_n^2}\frac{4P_o}{t_d}\left[\cos\omega_n\left(t-\frac{t_d}{4}\right) - \cos\omega_n t - \cos\omega_n(t-t_d) + \cos\omega_n\left(t-\frac{3t_d}{4}\right)\right]$$

15점

t_d 이후는 free vibration 이므로

$$u(t) = u(t_d)\cos\omega_n(t-t_d) + \frac{\dot{u}(t_d)}{\omega_n}\sin\omega_n(t-t_d)$$

$$u(t_d) = \frac{1}{m\omega_n^3}\frac{4P_0}{t_d}\left[\sin\omega_n\frac{3}{4}t_d - \sin\omega_n t_d + \sin\omega_n\frac{t_d}{4}\right]$$

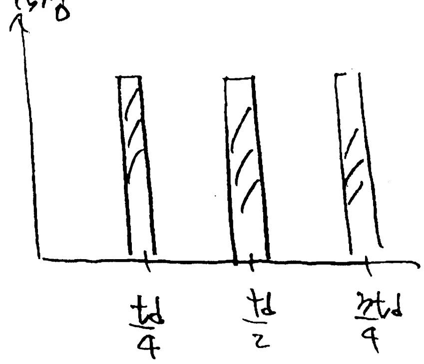
$$\dot{u}(t_d) = \frac{1}{m\omega_n^2}\frac{4P_0}{t_d}\left[\cos\omega_n\frac{3}{4}t_d - \cos\omega_n t_d - 1 + \cos\omega_n\frac{t_d}{4}\right]$$

20식

$$(u_{st})_0 = \frac{P_0}{k} = \frac{P_0}{\frac{k}{m}\cdot m} = \frac{P_0}{\omega_n^2 m}$$

$$\frac{p_0 t_d \pi}{4\pi^2 m}$$

(4) 15점



$I_1 = I_2 = I_3 = \frac{1}{2} \times p_0 \times \frac{t_d}{2} = \frac{P_o t_d}{4}$

2점

단위 Impulse에 의한 displacement

$h(t-\tau) = \frac{1}{m\omega_n} \sin[\omega_n(t-\tau)]$

5점

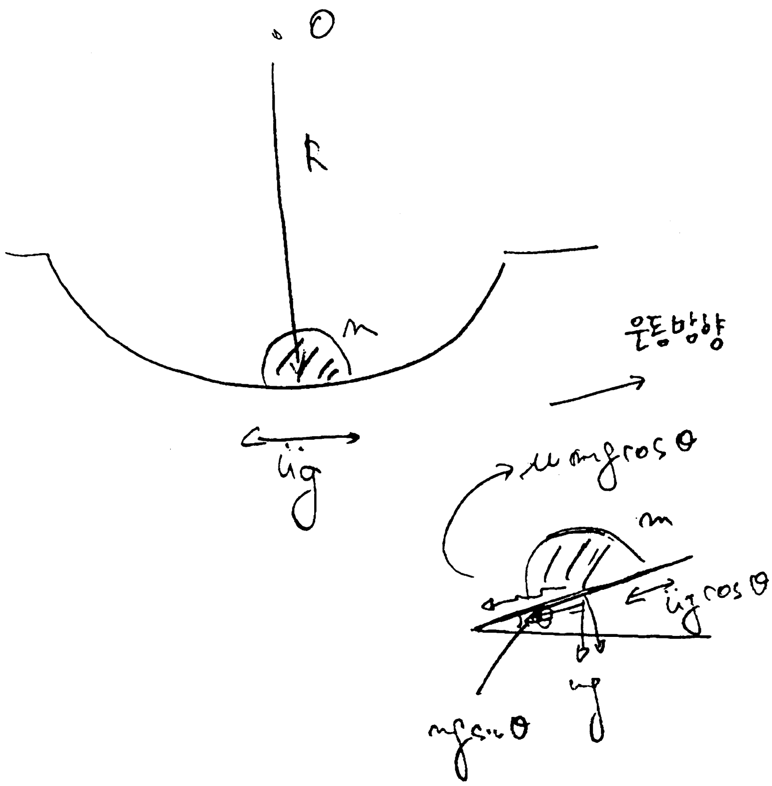
$u(t) = \frac{P_o t_d}{4} h(t - \frac{t_d}{4}) + \frac{P_o t_d}{4} h(t - \frac{t_d}{2}) + \frac{P_o t_d}{4} h(t - \frac{3}{4}t_d)$

$= \frac{P_o t_d}{4} \frac{1}{m\omega_n} \sin\pi(t - \frac{t_d}{4}) + \frac{P_o t_d}{4} \frac{1}{m\omega_n} \sin\pi(t - \frac{t_d}{2}) + \frac{P_o t_d}{4} \frac{1}{m\omega_n} \sin\pi(t - \frac{3}{4}t_d)$

$= \frac{P_o t_d}{4m\pi} \left[\sin\pi(t - \frac{t_d}{4}) + \sin\pi(t - \frac{t_d}{2}) + \sin\pi(t - \frac{3}{4}t_d) \right]$

15점

문제 4.
(25점)



가(10점)

점 O에 대한 Moment 평형

$$Rm(R\ddot{\theta} + \ddot{u}_g\sin\omega t\cos\theta) \pm R\mu mg\cos\theta + Rmg\sin\theta = 0$$

$$mR^2\ddot{\theta} + mR\ddot{u}_{g0}\sin\omega t\cos\theta \pm R\mu mg\cos\theta + Rmg\sin\theta = 0$$

θ가 매우 작을 때

$\cos\theta \fallingdotseq 1 \quad \sin\theta \fallingdotseq \theta$

$$mR^2\ddot{\theta} + mR\ddot{u}_{g0}\sin\omega t \pm R\mu mg + Rmg\theta = 0$$

$$mR^2\ddot{\theta} + Rmg\theta \pm R\mu mg = -mR\ddot{u}_{g0}\sin\omega t$$

5점

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{R\cancel{m}g}{\cancel{m}R^2}} = \sqrt{\frac{g}{R}}$$

$$\omega_n = \frac{2\pi}{T_n} \qquad T_n = 2\pi/\omega_n = 2\pi\sqrt{\frac{R}{g}}$$

10점

나.(5점)

$$mR^2\ddot{\theta} + Rmg\theta \pm \underbrace{R\mu mg}_{F} = \underbrace{-mR\ddot{u}_g \sin\omega t}_{p(t)}$$

$\omega = 2\pi/T_n = \omega_n$ 상황이 있을 때의 경우 $\quad \pi P_0\theta_0 < 4F\theta_0 \quad E_I < E_D \longrightarrow \frac{F}{P_0} > \frac{\pi}{4}$

$\frac{F}{P_0} > \frac{\pi}{4}$ 만족해야 함

3점

$$\frac{R\mu mg}{mR\ddot{u}_g} > \frac{\pi}{4}$$

$$\mu > \frac{\pi}{4} \times \frac{mR\ddot{u}_{g0}}{\cancel{R}mg} = \frac{\pi\ddot{u}_{g0}}{4g}$$

5점

다(10점) $\quad \omega = 5 \times 2\pi/T_n = 5\omega_n$ 에서 등가점성감쇠비 이용하여 결정

$$\zeta_{eq} = \frac{2}{\pi}\frac{1}{\omega/\omega_n}\frac{\theta_F}{\theta_0} = \frac{2}{\pi}\cdot\frac{1}{5}\times\frac{\mu}{u_0} = \frac{2\mu}{5\pi}\frac{1}{\theta_0}$$

$$\theta_F = \frac{F}{k} = \frac{P\mu mg}{Rmg} = \mu$$

3점

$$R_d = \frac{\theta_0}{(\theta_{st})_0} = \frac{1}{\sqrt{(1-(\omega/\omega_n)^2)^2 + (2\zeta_{eq}(\omega/\omega_n))^2}}$$

$$= \frac{1}{\sqrt{(-24)^2 + \left(\frac{4\mu}{\cancel{5}\pi}\frac{\cancel{5}}{\theta_0}\right)^2}} \qquad (\theta_{st})_0 = \frac{P_0}{k} = \frac{mR\ddot{u}_{g0}}{\cancel{R}mg} = \frac{\ddot{u}_{g0}}{g}$$

$$\left(\frac{\theta_0}{(\theta_{st})_0}\right)^2 = \frac{1}{(-24)^2 + \left(\frac{4\mu}{\pi\theta_0}\right)^2}$$

$$\left(\frac{g}{\ddot{u}_{g0}}\right)^2 \theta_0^2 \left((-24)^2 + \left(\frac{4\mu}{\pi}\right)^2\left(\frac{1}{\theta_0}\right)^2\right) = 1$$

$$\left(\frac{24g}{\ddot{u}_{g0}}\right)^2 \theta_0^2 + \left(\frac{g}{\ddot{u}_{g0}} \times \frac{4\mu}{\pi}\right)^2 = 1$$

$$\theta_0^2 = \frac{1 - \left(\frac{4\mu g}{\pi \ddot{u}_{g0}}\right)^2}{\left(\frac{24g}{\ddot{u}_{g0}}\right)^2}$$

$$\theta_0 = \frac{\ddot{u}_{g0}}{24g}\sqrt{\left(1 - \left(\frac{4\mu g}{\pi \ddot{u}_{g0}}\right)^2\right.}$$

10점