Engineering Mathematics I

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MID – EXAM

[Problem 1] (10 points) Find the inverse and the determinant of a general 3 x 3 matrix A = [a_{jk}] explicitly in terms of the a_{jk} 's.

[Problem 2] (10 points) Find an eigenbasis of the following matrix and diagonalize the matrix (Show the details).

$$\begin{bmatrix} -2.5 & -3 & 3 \\ -4.5 & -4 & 6 \\ -6 & -6 & 8 \end{bmatrix}$$

[Problem 3] (10 points) Consider the following matrix form of a linear system

$$Ax = b \,,$$

where $A \in R^{n \times n}$, $x \in R^n$, $b \in R^n$. Let $\hat{A}$ be the n x n matrix consisting of the first n column vectors of the row echelon form obtained at the end of Gauss elimination. Also, let $\hat{b}$ be the last column vector of the row echelon form. Then, prove by use of elementary matrices that if $\hat{x}$ is the solution of $\hat{A}\hat{x} = \hat{b}$, then $\hat{x} = x$.

[Problem 4] (20 points) Show that the inverse A^{-1} of an n x n matrix A exist if and only if none of the eingenvalues $\lambda_1, \cdots, \lambda_n$ of A is zero, and then A^{-1} has the eigenvalues $1/\lambda_1, \cdots, 1/\lambda_n$.

[Problem 5] (20 points) Consider 1^{st} order nonhomogeneous linear ODEs.

(a) Find the general solution of the following equation based on mathematically logical development

$$y' + p(x)y = r(x)$$

(b) Find the particular solution of the following equation using the formula obtained in 5(a).

$$y' - (1 + 3x^{-1})y = x + 2, \quad y(1) = e - 1.$$

[Problem 6] (10 points) Test for the exactness of the following equations. If exact, solve the equations. If not, use an integrating factor to solve the equations.

(a) $2y^{-1}\cos 2x dx = y^{-2}\sin 2x dy$

(b) $2xy dy = (x^2 + y^2)dx$

[Problem 7] (20 points) Consider the following 2nd order ODE

$$(1 - 2x - x^2)y'' + 2(1 + x)y' - 2y = 0.$$

(a) Show that $y_1(x) = x + 1$ is a solution of the equation.

(b) Find another solution of the equation using reduction of order (Show the details of your work and do not use any formula in the textbook).

(c) Show that two solutions are linearly independent.

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