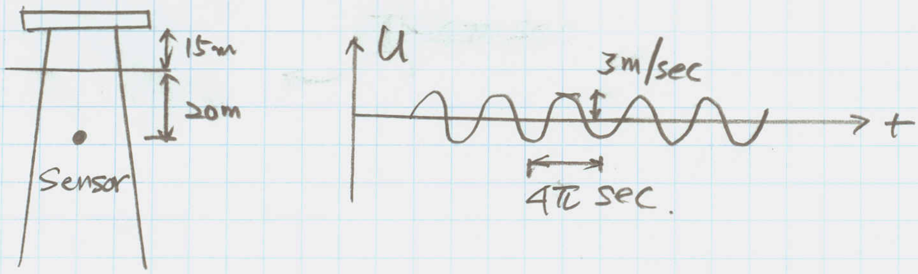
Mid-Term Solution

Oct. 30. 2007

1.



(1) $\omega = \frac{2\pi}{T} = 0.5$ rad/sec

In deep water ; $\omega^2 = gk$ or $\lambda = g\frac{2\pi}{\omega^2}$

$\Rightarrow \lambda = 251$ m ($k = 0.025$)

(2) $u_{max} = \frac{\partial \phi}{\partial x}\Big|_{max} = \omega A e^{kz}$

At $z = -20$, $u_{max} = 3$ m/sec

$\Rightarrow A = \frac{3}{\omega e^{kz}} = 10$ m. It doesn't hit the deck.

(3) $V_p = \frac{\omega}{k} = \sqrt{\frac{g}{k}}$ in deep water

$V_p = 20$ m/sec.

(4) $u_{max} = w_{max} = \omega A e^{kz}$

$\Rightarrow w_{max} = 3$ m/sec.

(5) $p = -\rho \frac{\partial \phi}{\partial t} - \rho g z$

$= \rho g [e^{kz} \eta - z]$

$\rho = 1025 \text{ kg/m}^3$ for sea water

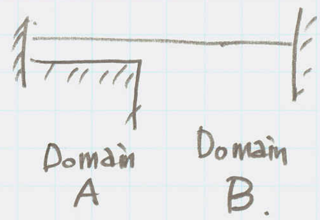
$\Rightarrow p = 1.025 \times 10^4 [6.065 \cos \omega t + 20]$

$\therefore p_{max} = 26.065 \times 1.025 \times 10^4$ N

$= 2.668 \times 10^5$ N.

(6) $\bar{E} = \frac{1}{2} \rho g A^2 = 5.125 \times 10^5 \text{ J/m}^2$

2. $\omega = \frac{2\pi}{T} = 0.575$ rad/sec.



- In domain B,

$\omega^2 = gk$ or $\lambda = g \frac{2\pi}{\omega^2} = 190$ m

- In domain A, start with the assumption of shallow depth. (Why? $3 \ll 190$)

$\omega^2 = g k^2 h$ or $\lambda = \sqrt{gh} \frac{2\pi}{\omega} = 60$ m

Since $\lambda \approx 20h$, the assumption of shallow depth is valid.

(1) • $\lambda = 190$ m in domain B

• $\lambda = 60$ m in domain A.

(2) ∘ $V_g = \frac{1}{2} V_p = \frac{1}{2}\frac{\omega}{k} = \frac{1}{2}\frac{g}{\omega} = 8.696$ m/sec. in domain B

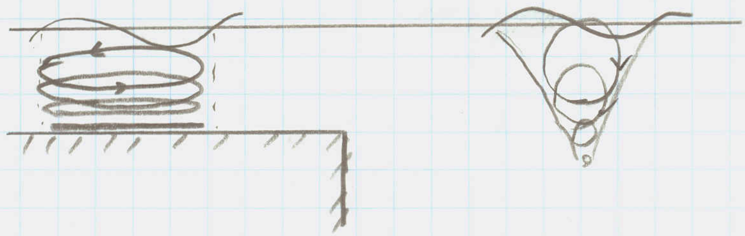
∘ $V_g = V_p = \sqrt{gh} = 5.477$ m/sec in domain A.

The first waves move with group velocities, so the times of returning become

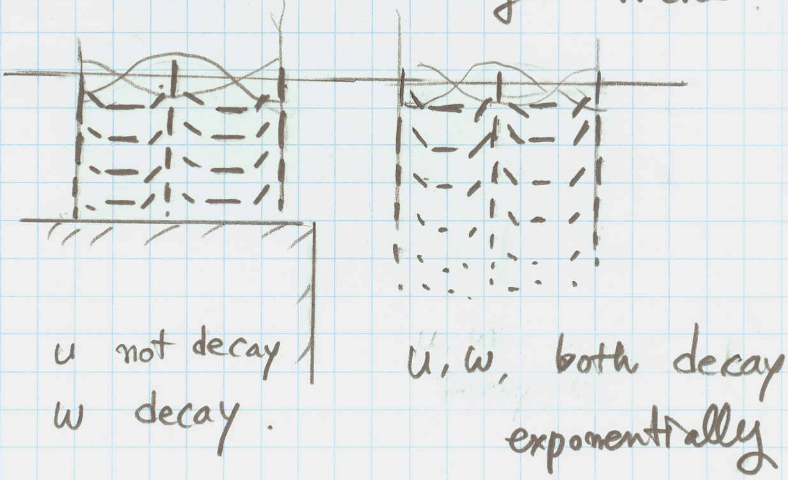
• domain B : $2 \times 500 / 8.696 = 115$ sec.

∘ domain A ; $2 \times 300 / 5.477 = 110$ sec.

(3) • Before reflection : Progressive waves



• After reflection : Standing waves



3. (1) ∘ $\frac{D}{Dt}(z-\eta)=0$ or $\frac{\partial\eta}{\partial t}+\frac{\partial\phi}{\partial x}\frac{\partial\eta}{\partial x}+\frac{\partial\phi}{\partial y}\frac{\partial\eta}{\partial y}-\frac{\partial\phi}{\partial z}=0$

∘ $\frac{\partial\phi}{\partial t}+\frac{1}{2}\nabla\phi\cdot\nabla\phi+g\eta+\frac{P_{atm}}{\rho}=C(t)$

or the following will be fine

$$\frac{\partial\phi}{\partial t}+\frac{1}{2}\nabla\phi\cdot\nabla\phi+g\eta=0$$

(2) Whatever you did, we will give proper score as long as the procedure is acceptable

(3) Dispersion relation describe the relation between the time characteristics and the space characteristics of waves. It reduces the parameters that we should know to define waves. ($k, \omega \to \omega$ or k only)

(4) Buckingham's π theorem

- Number of dependent variables : M
- Number of unknown variables : N

⇒ Number of Nondimensional variables

$= M - N$

3 Similarities ; (1) Geometric Similarity
(2) Kinematic "
(3) Dynamic "