Eng Math 2. Mid Term (10/31/2007)
(Closed book and note: 120 min.)

1. Prove the following equation [20 points].

$$\nabla\cdot(\vec{F} \times \vec{G}) = \vec{G}\cdot(\nabla \times \vec{F}) - \vec{F}\cdot(\nabla \times \vec{G})$$

2. Evaluate the following integral [20 points].

$$\iint_S \vec{F}\cdot\vec{n}\, dA$$

Where $F = [3xy^2, yx^2-y^3, 3zx^2]$
S: The surface of $x^2 + y^2 \leq 25, 0 \leq z \leq 2$

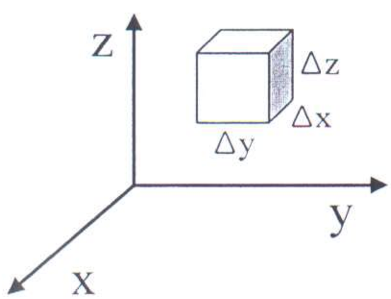
3. Find a Fourier transform of f(x) [15 points].

$$f(x) = \begin{cases} e^{2ix} & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

4. Prove the following equation [15 points].

$$F_c\{f'(x)\} = \omega F_s\{f(x)\} - \sqrt{\tfrac{2}{\pi}}\, f(0)$$

5. A liquid flows through the x-y-z space with a velocity of $\vec{\upsilon}$, as shown below. The density of liquid is ρ [30 points]
(1) Derive the continuity equation from mass balance.
(2) Derive the continuity equation for the incompressible fluid.



6. Find cases when the line integral below becomes path independent, and explain briefly each case. (20 pts, 5 pts each)

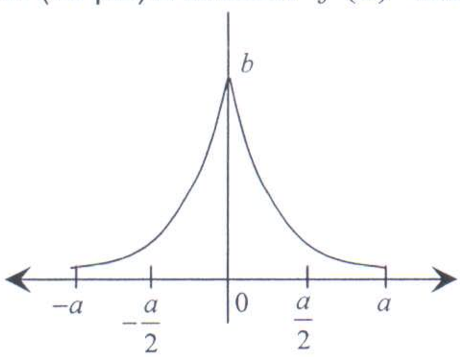
$$\int_{P_1}^{P_2} \vec{F}\cdot d\vec{r} \quad \text{where} \quad \vec{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}\,,\ \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

7. Answer to each question:

(a) (15 pts) Find the Fourier transform of 1. ($F(1) = ?$)

(b) (25 pts) Given that $F\left(\frac{1}{x^2+1}\right) = \pi e^{-w}$, Find the Fourier transform of $\frac{e^{2ix}}{9x^2+1}$. ($F\left(\frac{e^{2ix}}{9x^2+1}\right) = ?$)

8. (15 pts) If function $f(x)$ has the following form,



and $Z(x)$ is defined as follows

$$Z(x) = \sum_{n=-\infty}^{\infty} \delta(x - na).$$

Draw $f(x)\times Z(x)$, and $f(x) * Z(x)$, respectively. You should explain how you draw.

9. (25 pts) Evaluate the line integral, clockwise as seen by a person standing at the origin for the following F and C. Assume that the Cartesian coordinates to be right handed.

$$\vec{F} = \left[y, xy^3, -zy^3\right],\ \text{C: the circle } x^2 + y^2 = a^2,\ z = b\ (>0)$$

Have a great night!