## Electromagnetics

Final Exam | Date: June. 13, 2019 | Duration: 105 minutes | Full marks: 1,000 points

## Problem 1 (300 points)

(a) Derive analytically the TEM modes (i.e. $E \& H$-fields) propagating in $z$-direction in an infinite parallelplate transmission line (Assumption: infinite in extent in $x$-direction, two perfectly-conducting plates are separated by distance $d$ in $y$-direction and a dielectric of constitutive parameters $(\mu, \varepsilon)$ is filled in between. Hint: Start from Maxwell's Curl equations). (40 points)
(b) Q1: Derive the equations for $z$-dependent voltage and current on the lossless transmission line by using results obtained in (a) (Please define appropriate circuit elements per unit length that are comprised of constitutive parameters $(\mu, \varepsilon)$ and dimension of d [separation], w [width]). Q2: Find the solutions and express characteristic impedance and speed of signal propagation in terms of given parameters. ( 60 points)
(c) Q1: Assuming the transmission line is lossy (i.e. the conducting wall has finite conductivity and the dielectric has nonzero conductivity), build the "distributed-element" equivalent circuit model for infinitesimally short segment $(\Delta z)$ of the line and explain what each element represents. $\boldsymbol{Q} 2$ : Discuss whether there is any difference in the $E \& H$-fields in the lossy transmission line compared to those in the lossless counterpart. Q3: Under what conditions do we use a "distributed-element" model? (80 points) (d) Derive general transmission line equations from the circuit obtained in (c) (Hint: Use Kirchhoff's current and voltage laws). (40 points)
(e) $\boldsymbol{Q 1}$ : Is the lossy transmission line dispersive? Q2: Are voltage and current in such system in-phase or out-of-phase? Q3: How do these characteristics change as frequency varies? (40 points)
(f) $\boldsymbol{Q 1}$ : Derive the condition under which the lossy transmission line becomes distortion-less. $\boldsymbol{Q} 2$ : In such system, are voltage and current in-phase or out-of-phase? Please explain. (40 points)

## Problem 2 ( 50 points)

Find the input impedance of the finite-length transmission line at a distance $z$ ' from a load when the load is (a) purely resistive, (b) open-circuit, (c) short-circuit, (d) length is an integer multiple of quarter wave and (e) half wave (Denote the characteristic impedance of the line as $Z_{0}$ ).

## Problem 3 (50 points)

Provide analytical derivation of the standing wave ratio (SWR) for the lossless transmission line based on its definition. Discuss physical significance of SWR (e.g. when it does occur, how it is used, determining factors and so on).

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## Problem 4 (100 points)

The SWR on a $50(\Omega)$-lossless transmission line terminated in an unknown load impedance is found to be 3 . The distance between successive voltage maxima is $20(\mathrm{~cm})$, and the first maximum is located at $5(\mathrm{~cm})$ from the load.
(a) Determine $\Gamma$ and $Z_{\mathrm{L}}$. (40 points)
(b) Find the equivalent length $\left(l_{\mathrm{m}}\right)$ and terminating resistance $\left(R_{\mathrm{m}}\right)$, for two cases where $R_{\mathrm{m}}>50(\Omega)$ and $R_{\mathrm{m}}<50(\Omega)$, such that input impedance is equal to $Z_{\mathrm{L}}$. ( 60 points)

## Problem 5 (200 points)

(a) A d-c voltage $V_{0}$ is applied at $t=0$ directly to the input terminals on an open-circuited lossless line of length $l$. Sketch voltage and current waves on the line for the following time intervals: $0<t<T, T<t<2 T$, $2 T<t<3 T, 3 T<t<4 T, t>4 T$ where $T$ is time required for the signal to transverse the entire line. (100 points)
(b) As shown below, a lossless transmission line with a characteristic resistance $R_{0}$ is terminated at $z=l$ with a capacitor $C_{\mathrm{L}}$. A d-c voltage $V_{0}$ is applied to the line at $z=0$ through a series resistance $R_{0}$. When the switch is closed at $t=0$, plot $i_{\mathrm{L}}(t), v_{\mathrm{L}}(t)$ and the amplitude of the reflected wave $V_{1}^{-}(\mathrm{t})$ and $v\left(z, t_{1}\right)$ where $T<$ $t_{1}<2 T$, here $t_{1}$ is moment when the wave reflected at the load end is in the process of travelling toward the source. (100 points)


## Problem 6 (300 points)

(a) Discuss the working principle and advantage / disadvantage (if any) of quarter-wave transformer, single-stub and double-stub connections to the transmission line for impedance matching. ( 60 points)
(b) SWR of a $50(\Omega)$-lossless transmission line is measured to be 3 for signal of wavelength $=0.4(\mathrm{~m})$. The first voltage minimum appears at $z_{\mathrm{m}}{ }^{\prime}=0.05(\mathrm{~m})$ away from the load end. In that case, find $\Gamma, Z_{\mathrm{L}}, l_{\mathrm{m}}$ and $R_{\mathrm{m}}$ by using Smith Chart. (120 points) [Also, briefly describe the key steps in using the Smith Chart for this problem].
(c) A $50(\Omega)$-lossless transmission line is connected to a load impedance of $Z_{\mathrm{L}}=35-j 47.5(\Omega)$. Find the position and length of a short-circuited stub required to match the line by using Smith Chart. (120 points) [Also, briefly describe the key steps in using the Smith Chart for this problem].

