Electromagnetics

Midterm 1 | Date: Apr. 9, 2019 | Duration: 75 minutes | Full marks: 500 points

*** Follow the notation used in the textbook (**ε: permittivity, μ: permeability, ρ: charge density, σ: conductivity, ***E***, ***D***, ***B***, ***H*****:** electric field, electric flux, magnetic flux, magnetic field intensity vectors, ***J***: Volume current density vector). Note that the **Bold** characters denote the vector quantities.

Q1 (*130 points*)

(a) Write down Maxwell's equations and the two constitutive equations. (***20 points***)

(b) Derive general boundary conditions for the field quantities from Maxwell's equations. (***40 points***)

(c) Derive non-homogeneous wave equations for vector magnetic and scalar electric potentials under Lorentz condition. Assume that the wave exists in the simple medium (linear, isotropic, and homogeneous). (***30 points***)

(d) Suppose that there is time-varying charge density at the origin, which is the only source of the electromagnetic wave and is given by

$$\rho(\boldsymbol{R},t) = \rho_0 \delta(\boldsymbol{R}) \cos \omega t$$

Here, $\delta(\boldsymbol{R})$ denotes a Dirac-delta function specified in three-dimensional space.

Q: Determine the scalar electric potential V at a distance of $|\boldsymbol{R}|$ from the origin caused by $\rho(\boldsymbol{R},t)$ under Lorentz condition. (***20 points***)

Q: Describe the process how you will obtain the electric and magnetic fields by using the given electric potential [No derivation required!]. (***20 points***)

Q2 (*70 points*)

Suppose that the medium is source-free, simple (linear & isotropic) and lossless.

(a) Derive the homogeneous wave equation for the electric field. (***20 points***)

(b) Determine the solution to the equation obtained in (a) and briefly discuss its physical meaning. (***20 points***)

(c) Describe the relationship between electric field, magnetic field, and propagation direction of the electromagnetic wave obtained in (b). [*No derivation required!*] (***15 points***)

(d) For a simple medium whose constitutive parameters are not functions of time, one can derive the following rate equation from Maxwell's equations:

$$-\oint_S (\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{s} = \frac{\partial}{\partial t} \int_V \left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right) dv + \int_V \sigma E^2 \, dv.$$

Discuss the physical meaning of this rate equation. (***15 points***)

Q3 (*60 points*)

Suppose that there is a plane interface (*xy plane*) at $z = 0$ between free space ($z < 0$) and a perfect conductor ($z > 0$). There exists *time-varying* current flowing along such interface and is given by $\boldsymbol{J_s} = \boldsymbol{a_x} J_{s0} \cos \omega t$. Under the given condition, determine the instantaneous electric and magnetic field expressions in free space (i.e. at $z < 0$).

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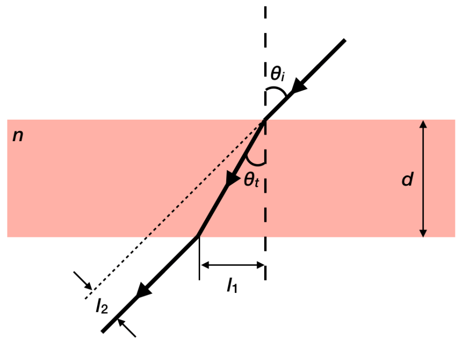
Q4 (***60 points***)

A light ray is incident from air obliquely on a transparent sheet of thickness d with an index of refraction n, as shown below. The angle of incidence is θ_i. Find θ_t, the distance l_1 at the point of exit, and the amount of the lateral displacement l_2 of the emerging ray.

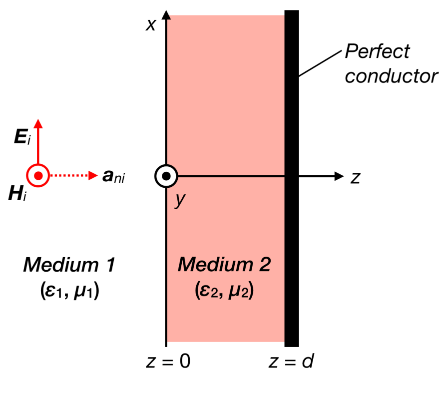


Q5 (***100 points***)

A uniform plane wave with

$$\boldsymbol{E_i}(x,z) = \boldsymbol{a_x} E_{i0} \cos\omega\left(t - \frac{z}{u_p}\right)$$

in medium 1 (ε_1, μ_1) is incident normally onto a lossless dielectric slab (ε_2, μ_2) of a thickness d backed by a perfectly conducting plane as shown below. Determine the thickness d that makes $\boldsymbol{E_1}(x,z)$ the same as if the dielectric slab were absent.



Q6 (***50 points***)

Discuss the physical meaning of the critical angle (**15 points**) and Brewster's angle (**35 points**) for the TE and TM waves in non-magnetic media.

Q7 (***30 points***)

Describe two examples how polarization of light is exploited in real-world applications.