Electromagnetics

Midterm 2 | Date: May. 14, 2019 | Duration: 80 minutes | Full marks: 750 points

*** Follow the notation used in the textbook (**ε: permittivity, μ: permeability, ***E***, ***D***, ***B***, ***H*****:** electric field, electric flux, magnetic flux, magnetic field intensity vectors). When determining electric and magnetic fields, please use the ***phasor*** notation (No instantaneous expressions required!). Unless specified, z is the default direction of propagation for the waveguides in any given coordinates.

Q1 (*100 points*)

A parallel-plate waveguide consists of *two perfectly conducting plates* that are parallel to each other. The conductors are separated by a dielectric medium characterized by (ε, μ) with thickness b. For simplicity, assume that the plates are infinite in extent in x-direction such that fields do not vary in that direction and the effect by fringing fields are neglected.

(a) Derive the electric and magnetic fields of the allowed modes for TE, TM, and TEM waves that *propagate* in the parallel-plate waveguide. Find the cut-off frequency for each case. ***(60 point)***

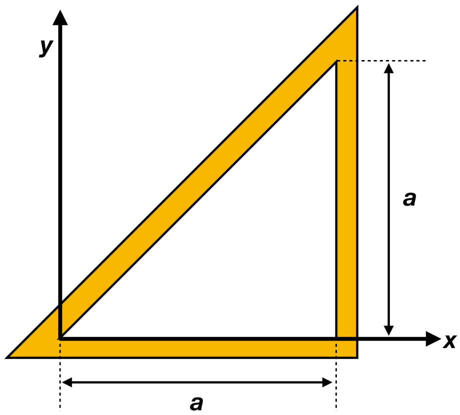
(b) Discuss the relationship of wave impedance *vs.* frequency for the allowed modes of the parallel-plate waveguides. ***(40 points)***

Q2 (*150 points*)

A waveguide is constructed so that the cross section of the guide forms a triangle with sides of length a, a, and $\sqrt{a}$ as shown below. The walls are *perfect conductors* and inside is air with (ε_0, μ_0).

(a) Determine electric and magnetic fields of the allowed modes for TE, TM and TEM waves *propagating* in the guide. ***(120 points)***

(b) If some modes are not allowed, explain why not. (*No derivation required, follow the logic used in the textbook*). ***(30 points)***



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Q3 (*200 points*)

A circular waveguide is a circular metal pipe having a uniform cross section of radius a. Inside is filled with a dielectric medium characterized by (ε, μ). Assume the metal pipe is a *perfect conductor* for simplicity.

(a) Derive the general expressions for electric and magnetic fields for TE and TM modes. ***(140 points)***

(b) Identify the cut-off frequencies for propagating TE and TM modes. What is the dominant mode for the circular waveguide? ***(60 points)***

Q4 (*200 points*)

A dielectric-slab waveguide of thickness d is situated in free space (ε_0, μ_0). A dielectric is characterized by (ε, μ). Assume that the guide sits on the xy plane and is infinite in extent in x-direction such that the fields do not vary in that direction and fringing fields are neglected.

(a) Derive the general expressions for electric and magnetic fields for the *propagating TM modes* in the dielectric waveguide. Explain under what condition and requirements the waves can propagate within the waveguide. ***(120 points)*** (*In the free space regions derive the fields for the only one side, i.e. either above or below the guide.*)

(b) Discuss the physical meaning of cut-off frequencies for the dielectric waveguide by comparing with those of other waveguide structures (parallel-plate and rectangular/circular waveguides). What factors do affect cut-off frequencies of these waveguides? ***(40 points)*** *(No derivation required!)*

(c) Transcendental equations for TM modes are given as below. If you want to send the TM-polarized waves with the frequency whose value coincides with the cutoff frequency of 2nd-order odd TM mode along the dielectric waveguide, how many TM modes are allowed at that frequency and what is the difference among such modes? (Note that an order of the mode starts from $n = 0$) ***(40 points)*** *(No derivation required!)*

$$\begin{cases} \dfrac{\alpha}{h_d} = \dfrac{\varepsilon_0}{\varepsilon_d}\tan\left(\dfrac{h_d d}{2}\right) & \cdots \text{ For odd TM modes} \\[2ex] \dfrac{\alpha}{h_d} = -\dfrac{\varepsilon_0}{\varepsilon_d}\cot\left(\dfrac{h_d d}{2}\right) & \cdots \text{ For odd TM modes} \end{cases}$$

(Here, α is the attenuation constant of the evanescent wave in free space and h_d is the wavenumber in the y-direction within the guide).

Q5 (*100 points*)

(a) Discuss the physical meaning of the resonant frequency in a rectangular cavity resonator. ***(30 points)***

(b) Is there power flow within the cavity? Why for your answer? ***(20 points)***

(c) What is the dominant mode for a rectangular cavity resonator? ***(50 points)***

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For your convenience, please use the following if needed.

1. Transverse field components expressed in terms of longitudinal components

$$\begin{cases} E_x^0 = -\dfrac{1}{h^2}\left(\gamma \dfrac{\partial E_z^0}{\partial x} + j\omega\mu \dfrac{\partial H_z^0}{\partial y}\right) \\ E_y^0 = -\dfrac{1}{h^2}\left(\gamma \dfrac{\partial E_z^0}{\partial y} - j\omega\mu \dfrac{\partial H_z^0}{\partial x}\right) \end{cases}, \quad \begin{cases} H_x^0 = -\dfrac{1}{h^2}\left(\gamma \dfrac{\partial H_z^0}{\partial x} - j\omega\varepsilon \dfrac{\partial E_z^0}{\partial y}\right) \\ H_y^0 = -\dfrac{1}{h^2}\left(\gamma \dfrac{\partial H_z^0}{\partial y} + j\omega\varepsilon \dfrac{\partial E_z^0}{\partial x}\right) \end{cases}$$

where $h^2 = \gamma^2 + k^2 \neq 0$ and $E_x(x,y,z) = E_x^0(x,y)e^{-\gamma z}$

2. Bessel function of the 1st kind of *n*-th order

$$J_n(hr) = \sum_{m=0}^{\infty} \frac{(-1)^m (hr)^{n+2m}}{m!(n+m)!2^{n+2m}},$$ where n is integer.

3. Bessel function of the 2nd kind of *n*-th order

$$Y_n(hr) = \frac{(\cos n\pi)J_n(hr) - J_{-n}(hr)}{\sin n\pi},$$ where n is integer.

4. Zeros of Bessel function of the 1st kind

Zeros for $J_n(x)$

p \ n	0	1	2	…
1	2.405	3.382	5.136	…
2	5.520	7.016	8.417	…
…	…	…	…	…

Zeros for $J'_n(x) = \partial[J_n(x)]/\partial x$

p \ n	0	1	2	…
1	3.382	1.841	3.054	…
2	7.016	5.331	6.706	…
…	…	…	…	…

Here, p denotes the order of the zero for the n-th order mode.