Advanced Thermodynamics (Final Exam.)

M2794.007900-001 **June 18, 2019**

1. If quantum numbers are large and the energy levels are closely located, what will be the density of quantum states, g(ϵ) for the particles moving in <u>one dimensional space</u> whose length is L? n indicate quantum numbers.

   Note: $n^2 = \left(\frac{8mL^2}{h^2}\right)\epsilon \equiv R^2$ 0 ——R——→ n_x

2. There are N indistinguishable particles in a piston of volume V. When the piston is heated and doing work simultaneously, describe what will be the system's new states in a statistical viewpoint.

3. Helium follows Maxwell-Boltzmann statistics. Find out the followings when the gas is confined in a space with volume of V.
   (1) Partition function (translation)
   (2) Internal energy
   (3) Entropy
   (4) Enthalpy
   (5) Chemical potential
   (6) Pressure
   (7) Heat capacity at constant volume

   cf. The density of the quantum states for translational motion is given by

   $g(\epsilon) = \frac{4\sqrt{2}\pi V}{h^3} m^{3/2}\epsilon^{1/2}$. If needed, use $\int_0^\infty \epsilon^{1/2} e^{-\epsilon/kT} d\epsilon = \frac{kT}{2}\sqrt{\pi kT}$.

   cf. $U = NkT^2\left(\frac{\partial \ln Z}{\partial T}\right)_V$.

4. Diatomic ideal gas is moving in <u>one-dimensional plane</u> which follows Maxwell-Boltzmann distribution. Find out the speed distribution $N(v)dv$, which implies the number of particles whose speed is in the range from v to $v + dv$. The number of particles is N, and the length of 1-dimensional plane is L.
   (※ Use the result in problem 1)

   cf. $\int_0^\infty \epsilon^{1/2} e^{-\epsilon/kT} d\epsilon = \frac{kT}{2}\sqrt{\pi kT}$, $\int_0^\infty \epsilon^{-1/2} e^{-\epsilon/kT} d\epsilon = \sqrt{\pi kT}$

5. One mole of oxygen and two moles of nitrogen are mixed in a constant temperature and pressure condition. If these gases follow Maxwell-Boltzmann distribution, what will be the entropy of mixing? Obtain the entropy of mixing by using a concept of <u>system partition function</u>.

6. For one-dimensional solid as an assembly of 2N distinguishable oscillators, half of them vibrate with a frequency, ν, and the other half with 2ν. Find out the followings.
(1) Partition function
(2) Internal energy, U.
(3) Heat capacity at constant volume, C_V.
(4) Helmholtz energy, F.
(5) Entropy, S.
(6) Find out the behavior of S at high temperature.

7. The total number of photons are not conserved, which means $\Sigma N_j = N$ can not be applied.
(1) Find out the number of photons for each quantum state.
(2) Find out the partition function, Z.
(3) Calculate the internal energy, U.
(4) Calculate the entropy, S.
(5) Calculate the Helmholtz energy, F.
(6) Calculate the pressure, P.
Note: $g(\nu)d\nu = \frac{8\pi V}{c^3}\nu^2 d\nu,\ \int_0^\infty \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15}.$

8. For Bosons, the equilibrium distribution is expressed as $f(\epsilon) = \frac{N(\epsilon)}{g(\epsilon)} = \frac{1}{e^{(\epsilon-\mu)/kT} - 1}$.
(a) Describe the procedure how to handle Bose-Einstein condensation phenomena.
(b) What will be the variation of the number of particles with temperature in the ground state and the excited state for Boson gases?

9. A system with two nondegenerate energy levels ε_0 and ε_1 ($\varepsilon_1 > \varepsilon_0 > 0$) is populated by N distinguishable particles at temperature T.
(1) Show that the average energy per particle is given by

$$u = \frac{U}{N} = \frac{\epsilon_0 + \epsilon_1 e^{-\beta\Delta\epsilon}}{1 + e^{-\beta\Delta\epsilon}},\ \Delta\epsilon = \epsilon_1 - \epsilon_0,\ \beta = 1/kT$$

(2) Show that when $T \to 0$

$$u \approx \epsilon_0 + \Delta\epsilon\, e^{-\beta\Delta\epsilon}$$

and when $T \to \infty$

$$u \approx \frac{1}{2}(\epsilon_0 + \epsilon_1) - \frac{1}{4}\beta(\Delta\epsilon)^2$$

(3) Show that the specific heat capacity at constant volume, c_v is

$$c_v = k\left(\frac{\Delta\epsilon}{kT}\right)^2 \frac{e^{-\Delta\epsilon/kT}}{(1 + e^{-\Delta\epsilon/kT})^2}$$

(4) Compute c_v in the limits $T \to 0$ and $T \to \infty$ and make a careful sketch of c_v versus $\triangle\varepsilon/kT$