Advanced Thermodynamics (Midterm Exam)

M2794.007900-001 **April 30, 2019**

1. Suppose a certain substance following Van Der Waals' equation of state which is $\left(P+\frac{a}{v^2}\right)(v-b)=RT$. Several isothermal lines of the substances in P-v graph is depicted below. The transition between the two different types of curves in an isothermal line is a curve having an inflection point **CP**, so-called critical curve and the inflection point is identical to a critical point. Above the critical point, the phase transition is continuously processed while the phase transition below the critical point undergoes vaporization or condensation. Find out its (1) critical temperature, (2) critical specific volume (3) critical pressure in terms of the constants a and b in the above Van Der Waals' EOS.

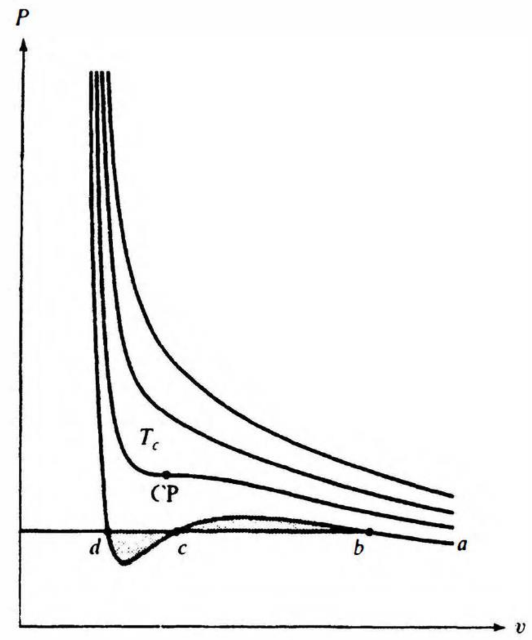


Figure Isotherms for a Van Der Waals' substance

2. Berthelot equation of state for real gases is given as $\left(P+\frac{a}{Tv^2}\right)(v-b)=RT$. Find out the followings in terms of thermodynamic properties and specific heat.

(1) Joule-Thomson coefficient $\mu_{JT}=\left(\frac{\partial T}{\partial P}\right)_h$

(2) Isentropic expansion coefficient $\mu_s=\left(\frac{\partial T}{\partial P}\right)_s$

(3) Using T-s diagram for nitrogen attached below, find out the final temperature when nitrogen is expanded from 300 K and 100 atm down to 10 atm in both isenthalpic and isentropic processes.

3. Show that

$$\mathrm{Tds} = c_p \mathrm{dT} - \mathrm{Tv}\beta \mathrm{dP}$$

for a reversible process, where the expansivity $\beta \equiv \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$.

4. The below figure shows a perfectly insulated container of which the inside is districted in half by diaphragm crossing the middle. One side is filled with air which consists of 80% nitrogen and 20% oxygen in equilibrium, whereas another side is vacuum. Assume the air is treated as an ideal gas. When rupturing the diaphragm, the inside air goes through so-called free expansion. A subscript of the initial state is allocated as 0 and that of the final state is 1. After free expansion, the final state reaches equilibrium state. Find out the changes of the following value in terms of the initial state variables (Note that the air should be regarded as an ideal gas mixture.)

(1) Temperature change

(2) Pressure change

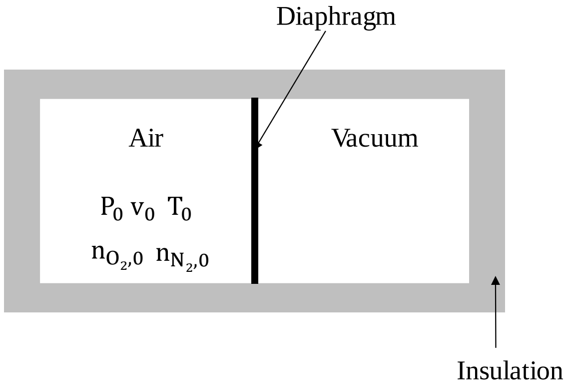
(3) Specific volume change

(4) Entropy change

(5) Enthalpy change

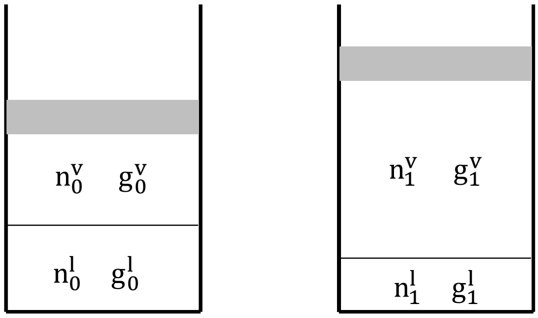
(6) Helmholtz function change

(7) Gibbs function change



5. A piston perfectly sealed is filled with liquid water and vapor in saturation state as the amount of "n" molecules as shown in the figure. Suppose a part of saturated liquid water turns to saturated vapor in reversible process. A subscript of the initial state is allocated as 0 and that of the final state is 1. Superscript "l" means liquid state and "v" means vapor.

(1) Show that $g^l = g^v$ (g is specific Gibbs function)



(2) Now, we define vaporization curve as shown in the figure. Assume latent heat "L" is independent of temperature and pressure and the vapor is treated as an ideal gas. The specific volume of saturated liquid is much lower than that of vapor so that can be relatively negligible. By using the above relation $g^l = g^v$, find out saturation pressure as a function of saturation temperature in vaporization curve $P_{sat} = f(T_{sat})$

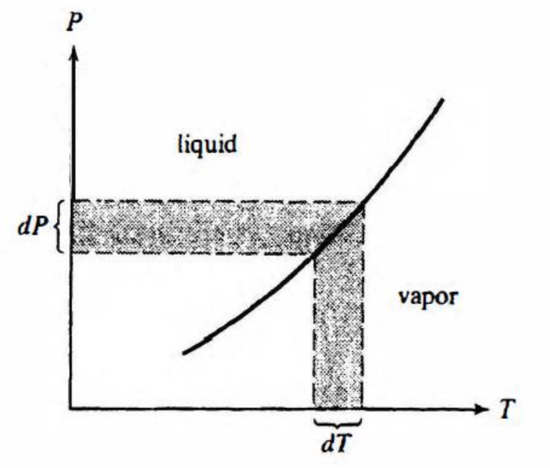


Figure Relationship between temperature and pressure for a liquid and vapor in equilibrium. The derivative $\frac{dP}{dT}$ is the slope of the vaporization curve.

6. The specific Gibbs function of a gas is given by

$$g = RT\ \ln\left(\frac{P}{P_0}\right) - A(T)P$$

where A is a function of temperature. Find out expressions for the followings.

(1) The equation of state (EOS)

(2) The specific entropy

(3) The specific Helmholtz function

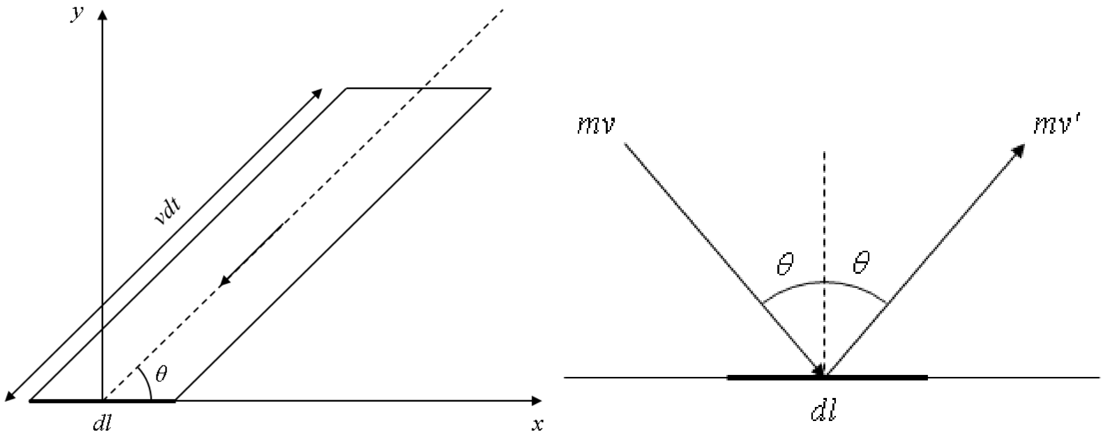
7. Express the chemical potential of an ideal gas in terms of the temperature T and volume V.

$$\mu = c_p T - c_v T \ln T - RT \ln V - s_0 T + a \quad \text{(a is constant)}$$

8. Suppose that molecules are moving in a two-dimensional space. The number of molecules per unit area is "n" and the mean speed of molecules is $\bar{v}$. Find out the followings.

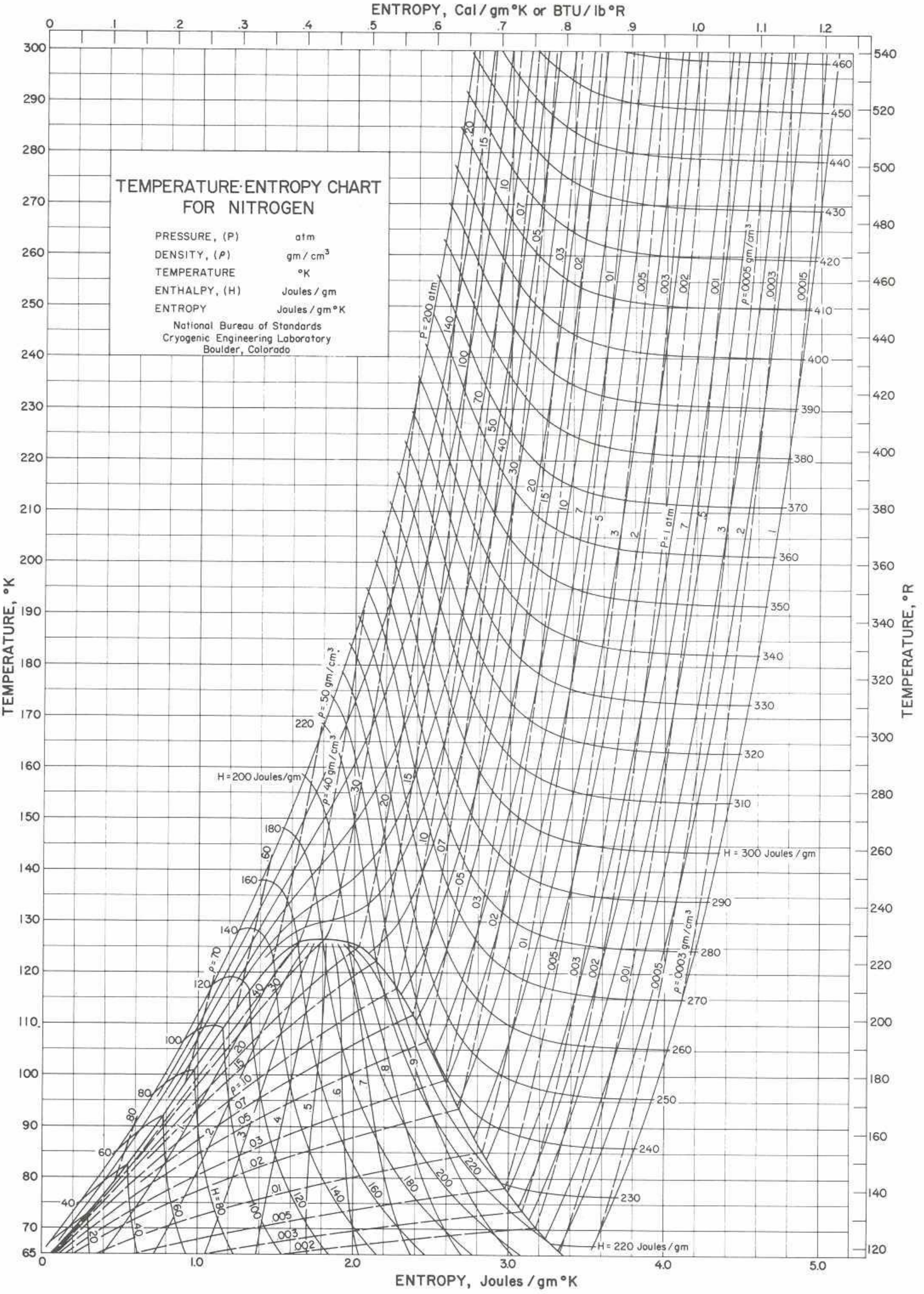
(1) Molecular flux, Φ (Number of collisions per unit time and unit length)

(2) Pressure of the gas, P



9. Assume "N" molecules are traveling in one-dimensional space. Find out a density $\rho = \frac{dN_v}{dv}$ as a function of temperature and velocity, which is known to be uniform for the same distance from the origin. dN_v represents the number of molecules whose speeds lie in the range v to $v + dv$. Evaluate all the unknown constants involved in the above formula.

Note: $\int_0^\infty e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$, $\int_0^\infty xe^{-ax^2} dx = \frac{1}{2a}$, $\int_0^\infty x^2e^{-ax^2} dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$, $\int_0^\infty x^3e^{-ax^2} dx = \frac{1}{2a^2}$, $\int_0^\infty x^4e^{-ax^2} dx = \frac{3}{8}\sqrt{\frac{\pi}{a^5}}$,


TEMPERATURE-ENTROPY CHART
FOR NITROGEN
PRESSURE, (P) atm
DENSITY, (ρ) gm / cm³
TEMPERATURE °K
ENTHALPY, (H) Joules / gm
ENTROPY Joules / gm °K
National Bureau of Standards
Cryogenic Engineering Laboratory
Boulder, Colorado
ENTROPY, Cal / gm °K or BTU / lb °R
ENTROPY, Joules / gm °K
TEMPERATURE, °K
TEMPERATURE, °R
P = 200 atm
P = 1 atm
ρ = 0005 gm/cm³
ρ = 50 gm/cm³
ρ = 40 gm/cm³
ρ = 0003 gm/cm³
H = 200 Joules/gm
H = 300 Joules / gm
H = 220 Joules / gm
H = 80