SEOUL NATIONAL UNIVERSITY
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

Closed book, closed note 10:30~11:45AM

Student ID \#: $\qquad$ Name: $\qquad$
[1] (15 points) Describe followings:
(1) Linear Dynamic Systems
(2) Mathematical model

| Problem <br> No <br> (points) | Points |
| :---: | :---: |
| $1(15)$ |  |
| $2(10)$ |  |
| $3(10)$ |  |
| $4(15)$ |  |
| $5(20)$ |  |
| $6(10)$ |  |
| Total(80) |  |

(3) Design and analysis
[2] (10 points) Solve following differential equation using Laplace Transformation.

$$
\begin{aligned}
& \ddot{y}+2 \dot{y}+4 y=1 \\
& y(0)=0, \dot{y}(0)=2
\end{aligned}
$$

[3] (10 points) Consider following mechanical system:

$\mathrm{m}=10 \mathrm{~kg}, \mathrm{k}=10 \mathrm{~N} / \mathrm{m}$.
(1) What is the natural frequency and damping ratio when $b=16 \mathrm{Ns} / \mathrm{m}$ ?
(2) "Sketch" and compare the time responses of the system when $b=16 \mathrm{Ns} / \mathrm{m}$ and $\mathrm{b}=10 \mathrm{Ns} / \mathrm{m}$.
[4] (15 points) Consider a system shown in Figure below. The radius and mass of the cylinder are $r$ and $m$, respectively. The stiffness of the spring is $k$. The displacement of the cylinder is defined as x . Assume no slip condition between the cylinder and floor.

(1) Obtain the equation of motion of the cylinder.
(2) Obtain state equation of this system.
(3) Compute natural frequency and sketch the motion of the cylinder, i.e., sketch the time response of the displacement of the cylinder for the following initial conditions:

$$
x(0)=1, \quad \dot{x}(0)=0 .
$$

[5] (20 points) Consider a quarter car vehicle suspension model shown in the Figure.
(1) Assuming that $z_{s}, z_{u}$, and $z_{r}$ are displacements of sprung mass, unsprung mass, and road from a reference, respectively, obtain a state equation with following definitions:

$$
\begin{aligned}
& x=\left[z_{s}-z_{u}, \dot{z}_{s}, z_{u}-z_{r}, \dot{z}_{u}\right]^{T} \\
& y=\left[\ddot{z}_{s}, z_{s}-z_{u}, z_{u}-z_{r}\right]^{T}
\end{aligned}
$$


(2) Assume that an actuator is placed between sprung and unsprung masses. In this case, the road displacement is considered as a disturbance, $d$, to the system and the actuator force is considered as a control input, $u$, to the system. Obtain a state equation of following form:

$$
\begin{aligned}
& \dot{x}=A x+B_{u} u+B_{d} d \\
& y=C x+D_{u} u+D_{d} d
\end{aligned}
$$


[6] (10 points) For the matlab m-file given below, sketch the result.
clear;
clc;
nит $=$ [7];
den=[10 15 60];
$t=0: 0.01: 10 ;$;
step(num,den,t)
grid on;
title(' step response' ,'fontsize',15);
xlabel(' t sec' ,'fontsize',15); ylabel( ‘ $y(t)$ ' ,'fontsize',15);

