SEOUL NATIONAL UNIVERSITY
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

SYSTEM ANALYSIS	**Spring 2008**
Final Exam **Closed book, closed note**	**Date: June 10, 2008 (tu)** **10:30~11:55AM**

Student ID #: ___Solution_____________ Name: ________________________

Problem No (points)	Points
1(20)	
2(15)	
3(20)	
4(10)	
5(20)	
6(10)	
Total(90)	

[1] (20 points) Describe followings:

(1) Linear dynamic systems

Systems : A combination of components acting together to perform a specific objective.

Dynamic : input-output relationship can be explained as a D.E.

$$y(t) \ \ \text{depends on} \ \ \{u(\tau)|\tau \le t\}$$

Linear : satisfies "Principle of Superposition"

SOLUTIONS

(2) Solution of the State equation: $\dot{x}(t) = Ax(t) + Bu(t), \ \ x(0) = x_0$

Definition : $x(t) = e^{A(t-t_0)} x_0 + \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau$

Free response, forced response

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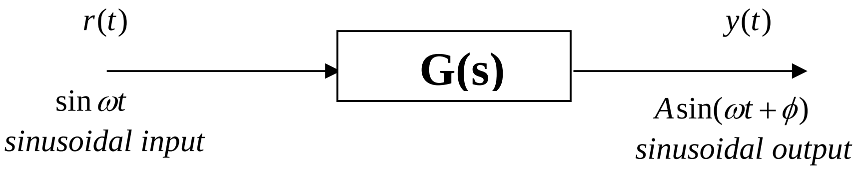
(3) Frequency response and bode plots

1.

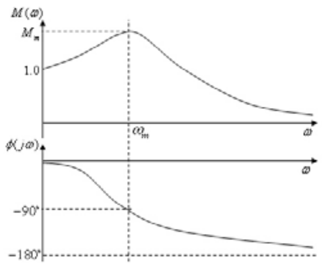


$$y(t) = A(j\omega)\sin(\omega t + \phi)$$

$$M(\omega) = \left|\frac{y(t)}{r(t)}\right| = |G(j\omega)| = A$$

$$\phi(j\omega) = \angle G(j\omega)$$

2.



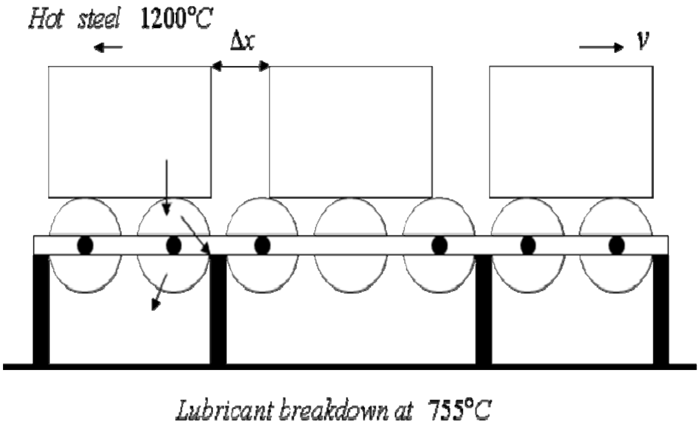
3. Lm G(jw) versus Log w

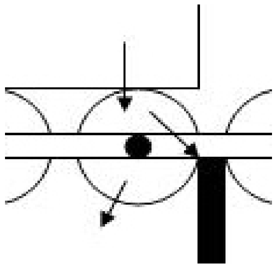
(4) System Design

Design : (system design)

-the process of finding a system that accomplishes a given task.

[2] (15 points) Consider a hot steel processing plant shown below. The temperature of the hot steel slab, T_s, is 1200 degree. Ambient air temperature, T_A, is 30 degree. The thermal resistance, R_1, for heat transfer between the hot steel slab and the roller is 10 $\frac{^{o}C}{kcal/s}$. The thermal resistance, R_2, for heat transfer between the roller and the ambient air is 20 $\frac{^{o}C}{kcal/s}$. The mass of the roller, m, is 5 Kg and the specific heat of the roller, C_p, is 1 $kcal/[kg\ ^{o}C]$. The lubricant breakdowns at 755 ^{o}C.



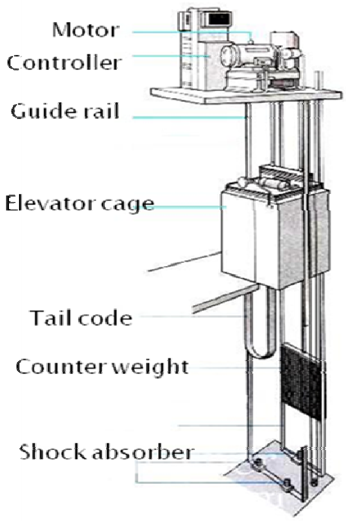


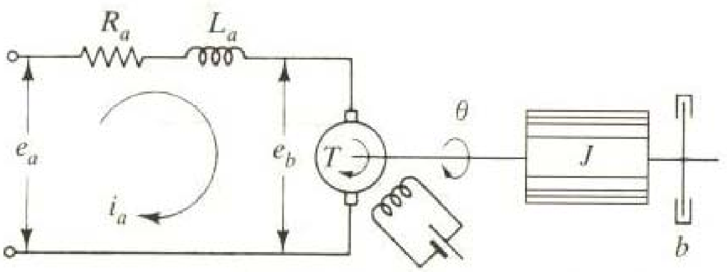
(1) Write down the heat balance equation for the roller.

(2) What is the steady state temperature of the bearing?

(3) What is your solution to avoid the breakdown of the lubricant of the bearing in the roller?

[3] (20 points) Consider an elevator system shown in the figure. The schematic diagram of the elevator system controlled by a DC electric motor is shown below. J is the equivalent moment of inertia of the elevator cage and the motor combined.





(1) Assuming that the armature inductance, L_a, is negligible, derive the equations of motion of the system.

$$e_b = K_b \frac{d\theta}{dt}$$

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$$

$$J \frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} = T = K\, i_a$$

(2) The position of the elevator cage, θ, can be controlled by the voltage input to the motor, e_a. Obtain the transfer function of the system, i.e., $G(s) = \frac{\Theta(s)}{E_a(s)}$.

$$e_b = K_b \frac{d\theta}{dt} \qquad\qquad K_b s\Theta(s) = E_b(s)$$

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a \quad \longrightarrow \quad (L_a s + R_a) I_a(s) + E_b(s) = E_a(s)$$

$$J \frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} = T = Ki_a \qquad\qquad (Js^2 + bs)\Theta(s) = T(s) = KI_a(s)$$

$$T.F = \frac{\Theta(s)}{E_a(s)} = \frac{K}{s(R_a Js + R_a b + KK_b)} = \frac{\frac{K}{R_a J}}{s\left(s + \frac{R_a b + KK_b}{R_a J}\right)}$$

$$= \frac{K_m}{s(T_m s + 1)} \qquad \begin{aligned} K_m &= K/(R_a b + KK_b) = motor\ gain\ constant \\ T_m &= R_a J/(R_a b + KK_b) = motor\ time\ constant \end{aligned}$$

(3) The velocity of the elevator cage is represented as

$$V_c = R\frac{d\theta}{dt}$$

where R is the equivalent radius of the elevator cage roller. The maximum voltage input of the motor, $e_{a,\max}$, is 300 Volts. What is the maximum speed of the elevator cage?

(4) Assuming that the transfer function between the voltage input to the motor and the angular position of the elevator cage is as follows:

$$\frac{\Theta(s)}{E_a(s)} = \frac{K_m}{s(T_m s + 1)}$$

where

$$K_m = K/(R_a b + KK_b) = motor\ gain\ constant$$

$$T_m = R_a J/(R_a b + KK_b) = motor\ time\ constant$$

The equivalent radius of the elevator cage roller, R, is 2 meter. The maximum voltage input of the motor, $e_{a,\max}$, is 300 Volts. What is the motor gain constant we need to have so that the maximum speed of the elevator is $30m/\sec$?

[4] (10 points) What are the characteristic parameters for the first order system and second order systems?

Time constant,

Damping ratio and natural frequency

[5] (10 points) Consider a mechanical system shown in the Figure. Obtain the steady-state outputs $x_1(t)$ and $x_2(t)$ when the input $p(t)$ is a sinusoidal force given by
$p(t) = P \sin \omega t$

[6] (10 points) Consider a system described by following state equation:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

where

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}, \text{B=}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \text{ and C=[0 1 0].}$$

The initial condition is: $x(0) = [0\ 2\ 0]^{\mathrm{T}}$ **.**

Compute the output for unit step input.

$$\begin{cases} \dot{x}_3 = 2x_3 + u \\ \dot{x}_2 = x_2 + 3x_3 \\ \dot{x}_1 = x_1 + x_2 + 2x_3 \end{cases}, \quad u(t) = 1(t), \quad \mathrm{x}(0) = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\mathrm{x}_3(t) = e^{2t}\,\mathrm{x}_3(0) + \int_0^t e^{2(t-\tau)} u(\tau) d\tau$$

$$= e^{2t} \cdot \int_0^t e^{-2\tau} d\tau = e^{2t}\left(-\frac{1}{2}\right)\left(e^{-2t} - 1\right) = -\frac{1}{2}\left(1 - e^{2t}\right)$$

$$\mathrm{x}_2(t) = e^{t}\,\mathrm{x}_2(0) + \int_0^t e^{(t-\tau)} 3\mathrm{x}_3(\tau) d\tau$$

$$= e^{t}\,\mathrm{x}_2(0) + \int_0^t 3e^{t}\left(-\frac{1}{2}\right)\left(e^{-\tau} - e^{\tau}\right) d\tau$$

$$= 2e^{t} - \frac{3}{2}e^{t}\left\{-\left(e^{-t} - 1\right) - \left(e^{t} - 1\right)\right\} = -e^{t} + \frac{3}{2} + \frac{3}{2}e^{2t}$$

$$y(t) = \mathrm{x}_2(t)$$