Inviscid Flow
Final Exam
2019

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1. (10 points) Explain the "dispersion" phenomenon in the gravity-driven surface wave.

2. (20 points) Oscillation of air bubble entrained in the wake of marine vessel is a major noise source, which is important for applications such as acoustic ship detection. The flow past an oscillating small bubble (radius $a(t)$), which is assumed to be spherical, can be modeled by superposing uniform flow, a point doublet (strength μ), and a point source (or sink) (strength m), located at the origin. Find the expression for μ and m such that they denote the oscillating bubble, i.e., bubble radius is changing in time.

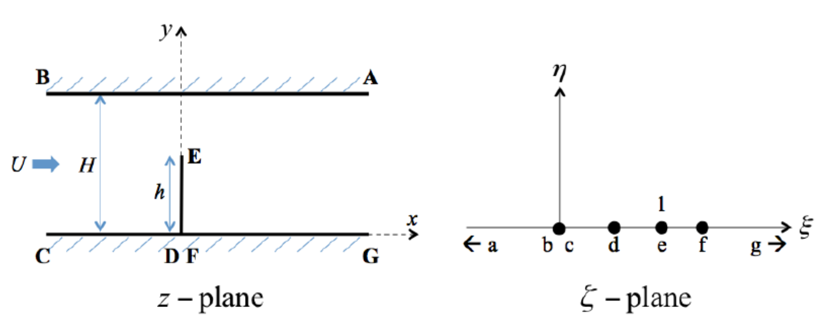
3. (30 points) Consider a three-dimensional potential flow generated by the combination of the freestream (U_∞) and a point source (strength Q) positioned at the origin. Use the polar (r, θ) coordinates. Given that the surface of the body in this flow has a stream-function value of zero, derive the equation for the surface shape in terms of r and θ. Also find the expression for the pressure at the stagnation points, if any.

4. (40 points) Consider a Schwartz-Christoffel transformation of a channel flow with a vertical plate extending part way (in z-plane) across to the upper half of the ς-plane. Let's assume that the points are to be mapped as indicated (e.g., A to a) in the figure below. The velocity far upstream in the channel is U, and the channel has a uniform height H, except at the plate, which is of height h.

   (a) (10 points) Obtain the differential equation, $dz/d\zeta$, which defines the mapping. You don't need to integrate the equation to get the explicit form of the mapping function.

   (b) (15 points) Determine the complex potential in the ς-plane, $F(\varsigma)$.

   (c) (10 points) Determine the constant K in the differential equation in (a) and also a relationship between the unknown values of d and f. Use the proper boundary conditions. You may have to assign a negative value to the square root of a positive quantity to obtain the relation between d and f.



5. (30 points) Let's consider a small-amplitude plane wave is traveling along the liquid surface with velocity c with the liquid depth of h. The governing equation for velocity potential (ϕ) and associated linearized boundary conditions are as follows.

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

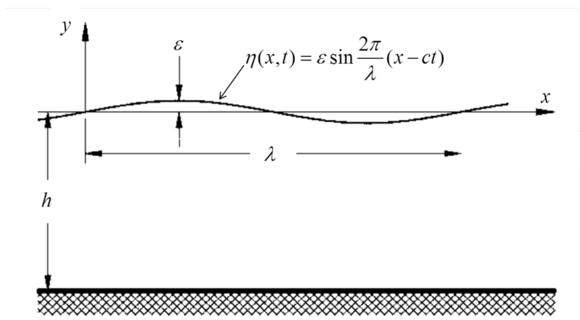
$$\frac{\partial\eta}{\partial t}(x,t) = \frac{\partial\phi}{\partial y}(x,0,t),\ \ \frac{\partial^2\phi}{\partial t^2}(x,0,t) + \frac{1}{\rho}\frac{\partial P(x,t)}{\partial t} + g\frac{\partial\phi}{\partial y}(x,0,t) = 0\ \ @\ y=\eta$$

$$\left.\frac{\partial\phi}{\partial y}\right)_{y=-h} = \frac{\partial\phi}{\partial y}(x,-h,t) = 0\ \ @\ y=-h$$

(a) (15 points) Explain the physical meaning of each boundary conditions.

(b) (15 points) Let's assume the profile of the surface wave as $\eta(x,t) = \varepsilon\sin\frac{2\pi}{\lambda}(x-ct)$. Then, the solution of given Laplace equation can be assumed to be the form given below. Derive the dispersion relation and find the values of dimensionless propagation speed (c^2/gh) for shallow and deep liquids conditions, respectively. (Note: you can ignore the effect of surface tension.)

$$\phi(x,y,t) = \cos\frac{2\pi}{\lambda}(x-ct)\left(C_1\sinh\frac{2\pi y}{\lambda} + C_2\cosh\frac{2\pi y}{\lambda}\right)$$



APPENDIX

In three-dimensional potential flow (in spherical coordinates),

stream function for a uniform flow: $\frac{1}{2}U_\infty^2 r^2\sin^2\theta$

for a source located at the origin: $\phi = -\frac{Q}{4\pi r}$, $\psi = -\frac{Q}{4\pi}(1+\cos\theta)$

for a doublet located at the origin: $\phi = \frac{\mu}{4\pi r^2}\cos\theta$, $\psi = -\frac{\mu}{4\pi r}\sin^2\theta$