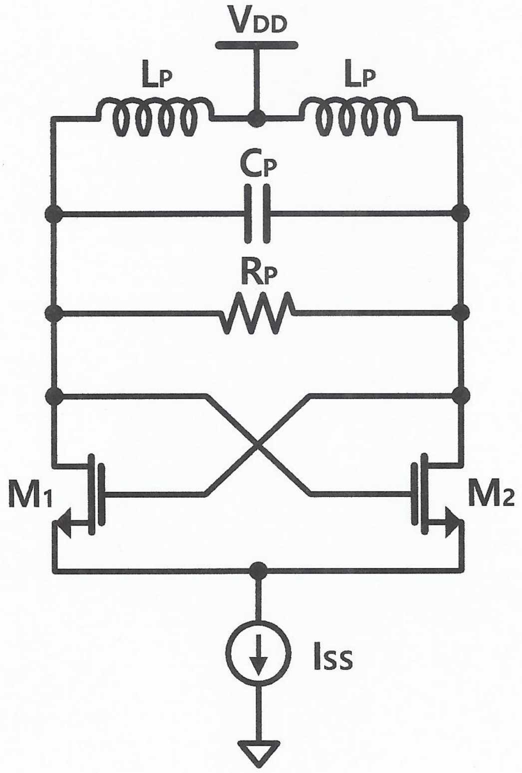
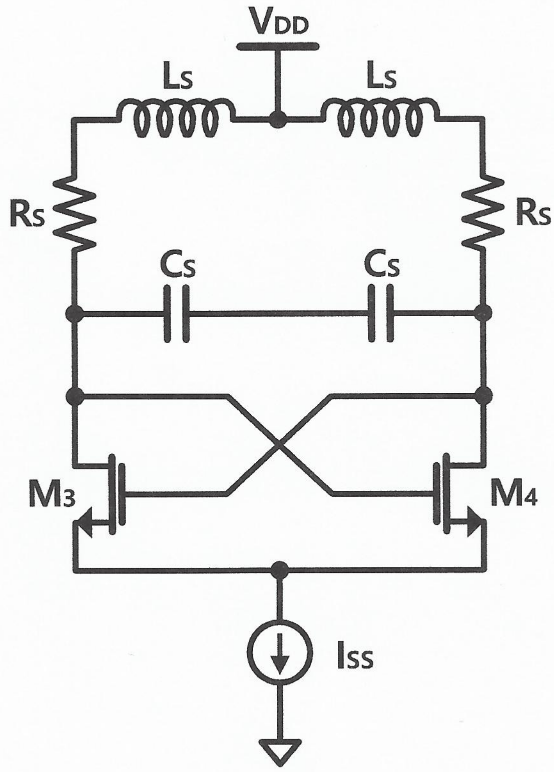
[1] Answer the following questions. Assume all the transistors are identical and $r_o = \infty$.

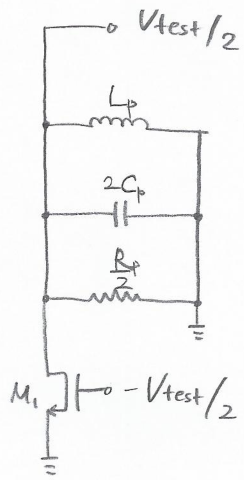


Circuit A

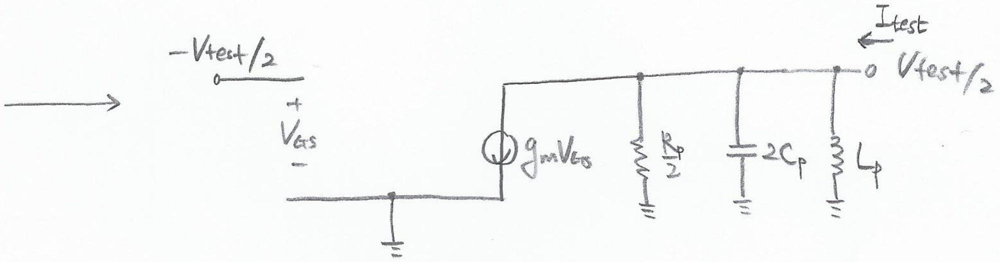


Circuit B

A. Find the oscillation frequency of the LC oscillator shown in Circuit A.



(Half-Circuit Analysis)



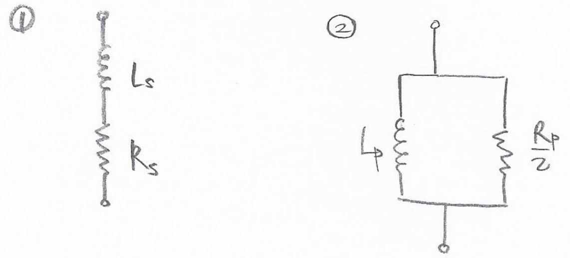
$$I_{test} = \frac{1}{2}V_{test}\left[-g_m + \frac{2}{R_p} + 2sC_p + \frac{1}{sL_p}\right]$$

$$\rightarrow \frac{V_{out}}{I_{out}} = \frac{\frac{1}{2}V_{test}}{I_{test}} = \frac{1}{-g_m + \frac{2}{R_p} + 2sC_p + \frac{1}{sL_p}} = \frac{R_pL_ps}{R_p + (2L_p - g_mR_pL_p)s + 2R_pL_pC_ps^2}$$

$\therefore$ oscillation frequency $\omega_0 = \sqrt{\frac{1}{2L_pC_p}}$

B. Determine the values of R_S, L_S, and C_S in terms of R_P, L_P, and C_P so that the impedance of Circuit B is the same as Circuit A. Use the oscillation frequency found from A. (Hint : Assume $R_p/2 \gg wL_p$)

$C_s = 2C_p$이고, ①과 ②가 서로 equivalent 하면 impedance가 서로 같아진다.



① 의 impedance $Z_s = R_s + j\omega L_s$

② 의 impedance $Z_p = j\omega L_p \parallel \frac{R_p}{2} = \frac{1}{\omega^2 L_p^2 + \left(\frac{R_p}{2}\right)^2}\left\{\omega^2 L_p^2 \left(\frac{R_p}{2}\right) + j\omega L_p \left(\frac{R_p}{2}\right)^2\right\}$

$$\approx \frac{\omega^2 L_p^2}{\left(\frac{R_p}{2}\right)^2} + j\omega L_p \qquad (\because R_p/2 \gg \omega L_p)$$

$\Rightarrow L_s = L_p$, $R_s = \frac{2\omega^2 L_p^2}{R_p}$ 이면 ① 과 ②의 impedance 가 서로 같아진다.

$$\therefore \begin{cases} R_s = \frac{2\omega^2 L_p^2}{R_p} = \frac{L_p}{R_p C_p} \quad \left(\omega = \sqrt{\frac{1}{2L_p C_p}} \text{ 대입}\right) \\ L_s = L_p \\ C_s = 2C_p \end{cases}$$

C. Determine whether the LC oscillator oscillates when g_m=1mS, R_S=1Ω, and L_S=1uH, and C_S=1nF.

A의 식으로부터 $2L_p - g_m R_p \cdot L_p \le 0$ 일 때, oscillation 하게 된다.

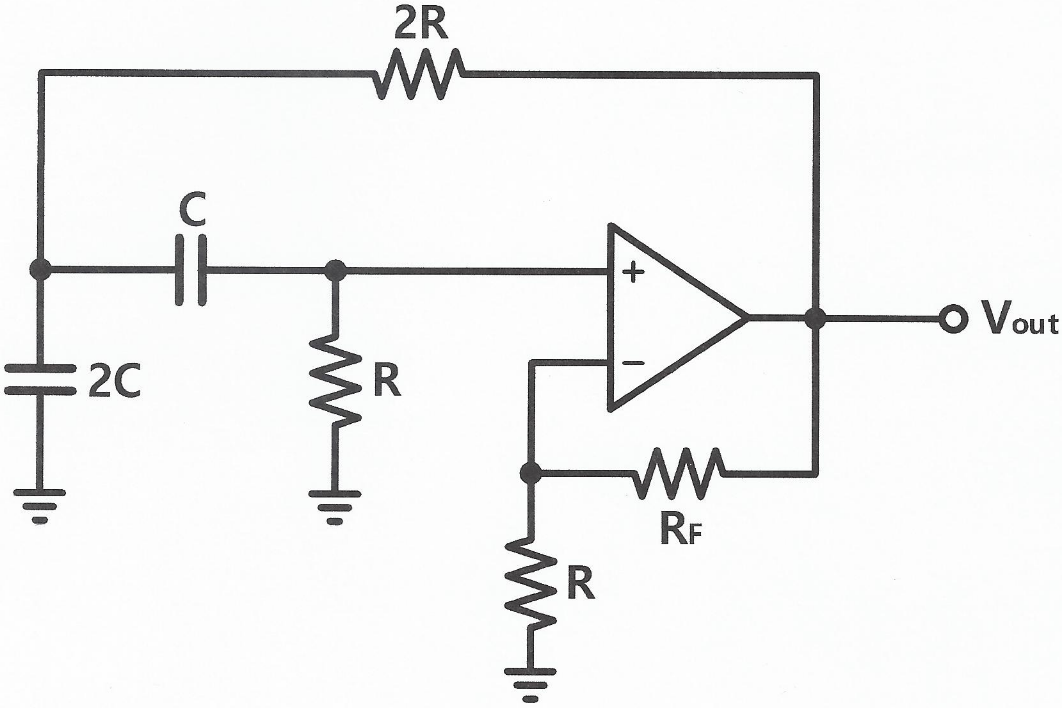
$(\Leftrightarrow g_m R_p \ge 2)$

B로부터 $R_p = \frac{L_p}{R_s C_p} = \frac{2L_s}{R_s \cdot C_s}$

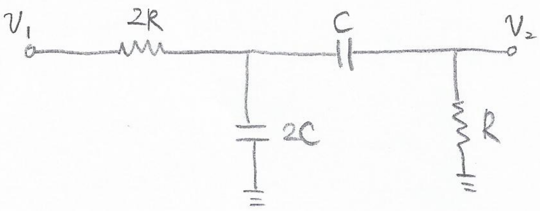
$$= \frac{2\times10^{-6}}{1\times10^{-9}} = 2\times10^{3}\ (\Omega)$$

$g_m R_p \ge 2$ 를 만족하므로 oscillation 한다.

[2] Answer the following questions. Assume the amplifier is ideal.



A. Derive the transfer function of the loop gain.



$$\frac{V_2}{V_1} = \frac{\frac{1}{2sC} \parallel \left(\frac{1}{sC}+R\right)}{2R + \frac{1}{2sC} \parallel \left(\frac{1}{sC}+R\right)} \cdot \frac{R}{\frac{1}{sC}+R}$$

$$= \frac{\frac{1+sRC}{(3+2sRC)sC}}{2R + \frac{1+sRC}{(3+2sRC)sC}} \cdot \frac{sRC}{1+sRC}$$

$$= \frac{1+sRC}{4s^2R^2C^2 + 7sRC + 1} \cdot \frac{sRC}{1+sRC} = \frac{RCs}{4R^2C^2s^2 + 7RCs + 1}$$

$\therefore$ Loop gain $H(s) = \frac{V_2}{V_1} \cdot \left(1 + \frac{R_F}{R}\right)$

$$= \left(1+\frac{R_F}{R}\right) \cdot \frac{RCs}{4R^2C^2s^2 + 7RCs + 1}$$

B. Find the minimum value of R_F for starting oscillation. Assume R=1kΩ, C=1uF.

positive feedback이므로 $\angle H(s) = 0°$ (or $360°$) 인 ω_0를 구하면,

$\omega_0 = \frac{1}{2RC}$ 일 때, $\angle H(s) = 0°$를 만족한다.

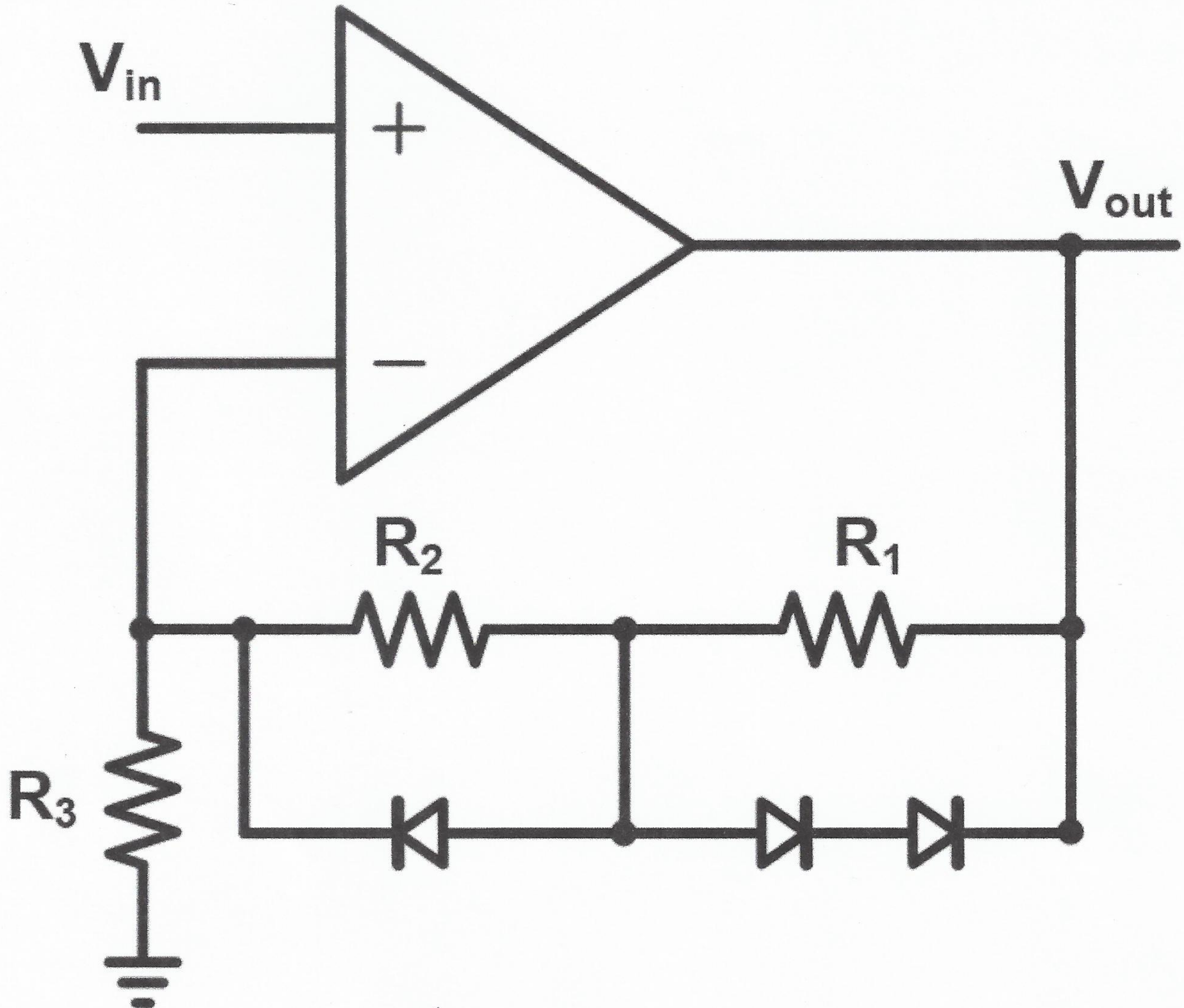
oscillation을 위해서는 $|H(j\omega_0)| \geq 1$ 의 조건을 만족해야 한다.

$$|H(j\omega_0)| = \left| \left(1 + \frac{R_F}{R}\right) \cdot \frac{1}{7} \right| \geq 1$$

$$\Leftrightarrow \quad R_F \geq 6R$$

$\therefore$ oscillation을 위한 최소 $R_F = 6R = 6k\Omega$

[1] Draw a $V_{in} - V_{out}$ curve of the circuit below. Specify the slope, condition of V_{in} or V_{out} when the diode turned on. (The voltage when the diode is turned on is $V_{D,ON}$)



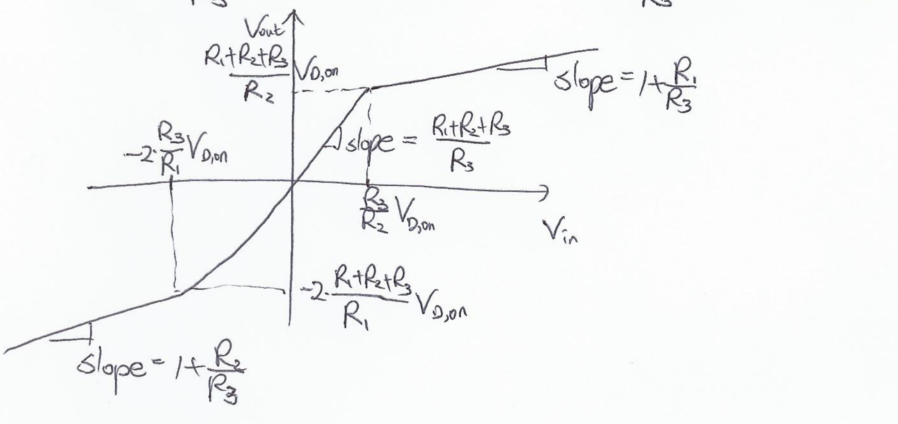
i) Diode off $\quad V_{out} = \frac{V_{in}}{R_3}(R_1+R_2+R_3) = \left(1+\frac{R_1+R_2}{R_3}\right)V_{in}$

ii) Diode on, $V_{in}>0$ $\quad \frac{V_{in}}{R_3}\cdot R_2 > V_{D,on} \rightarrow V_{in} > \frac{R_3}{R_2}V_{D,on}$

$$V_{out} = \frac{V_{in}}{R_3}(R_1+R_3)+V_{D,on} = \left(1+\frac{R_1}{R_3}\right)V_{in}+V_{D,on}$$

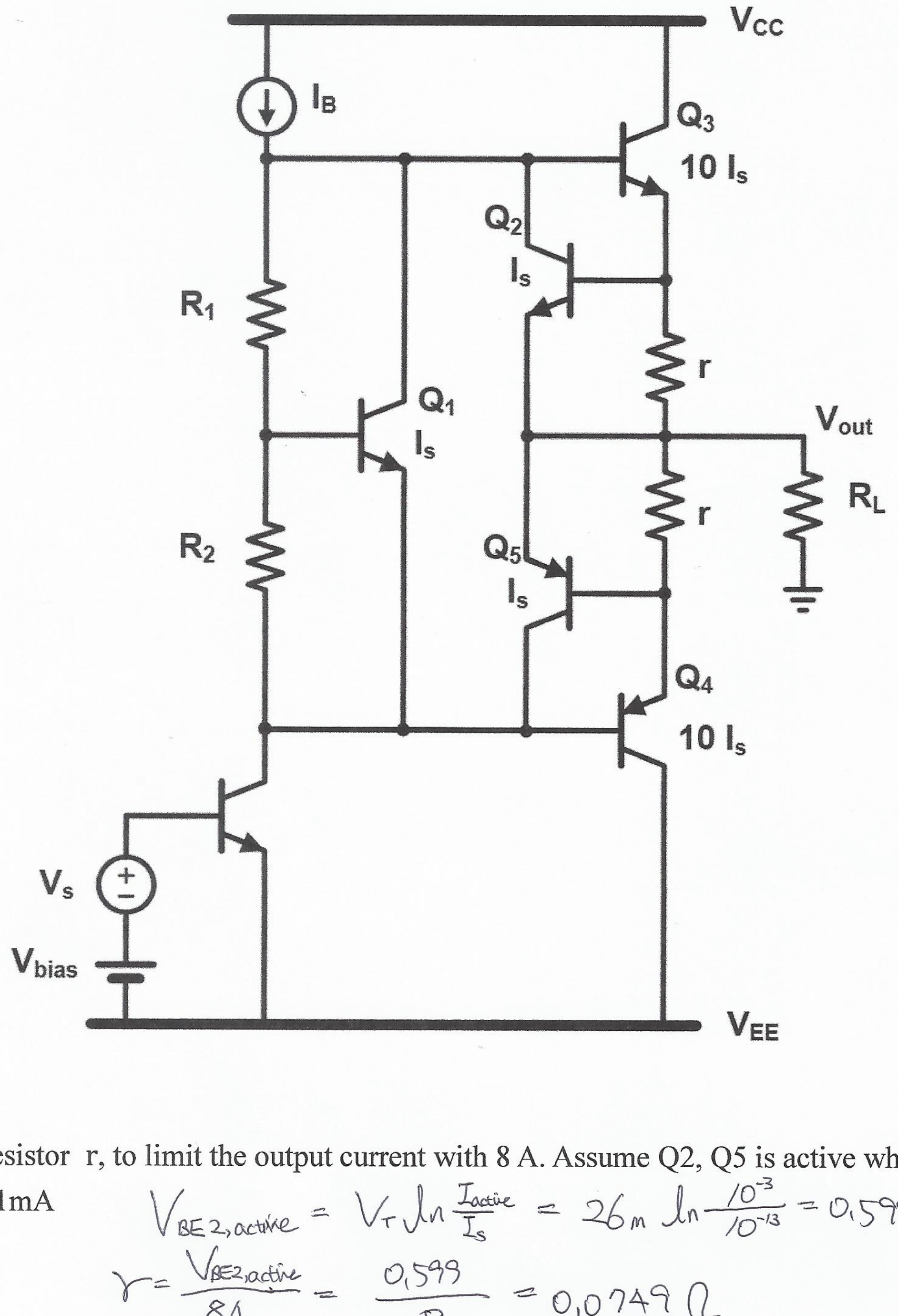
iii) Diode on, $V_{in}<0$ $\quad -\frac{V_{in}}{R_3}\cdot R_1 > 2V_{D,on} \rightarrow V_{in} < -2\frac{R_3}{R_1}V_{D,on}$

$$V_{out} = \frac{V_{in}}{R_3}(R_3+R_2)-2V_{D,on} = \left(1+\frac{R_2}{R_3}\right)V_{in}-2V_{D,on}$$



[2] Consider the following output stage circuit. (Assume $V_A = \infty$)

$V_T = 26\text{mV},\ I_s = 1 \times 10^{-13} A,\ R_L = 10\Omega,\ V_{CC} = -V_{EE} = 42\text{V}.$



A. Find the resistor r, to limit the output current with 8 A. Assume Q2, Q5 is active when collector current is 1mA

$V_{BE2,active} = V_T \ln \frac{I_{active}}{I_S} = 26m \ln \frac{10^{-3}}{10^{-13}} = 0{,}599\ V$

$r = \frac{V_{BE2,active}}{8A} = \frac{0{,}599}{8} = 0{,}0749\ \Omega$

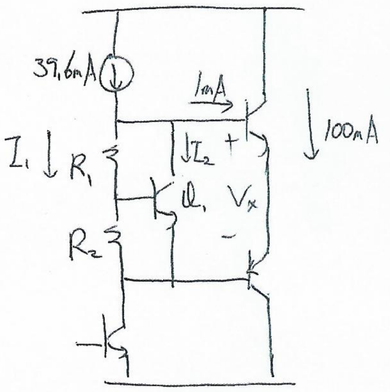
B. Find the minimum bias current I_B to deliver 80W power on 10Ω load with sinusoidal signal. Assume $\beta = 100$.

$V_{out} = V_p \sin wt$

$\frac{1}{2}\frac{V_p^2}{R_L} = 80 \rightarrow V_p = 40V$

$(1+\beta) I_B \geq \frac{40V}{10\Omega}$ $\quad \therefore I_B \geq 39{,}6mA$

C. Find the R_1 using the bias current I_B of problem B, when the Quiescent current at the output is 100mA. (Quiescent current is the current when it is producing an output of zero, $V_{out} = 0$) Assume $R_2 = 1k\Omega$, r = 0Ω, and β of Q_1 is big enough, that is, $I_{C1} \approx I_{E1}$.)



$$V_X = 2\cdot V_T \ln \frac{100mA}{10 I_S} = 1.5205V$$

$$V_{BE,Q_1} = V_T \ln \frac{I_2}{I_S} = 26mV \cdot \ln \frac{38.6mA - I_1}{10^{-13}}$$

$$\cancel{38.6mA = I_1 + I_2} = \quad I_1 = \frac{V_{BE,Q_1}}{R_2} = \frac{26m}{1k} \ln \frac{38.6m - I_1}{10^{-13}}$$

$$I_1 \simeq 693\mu A$$

$$I_1 (R_1 + R_2) = V_X = 693\mu A (R_1 + 1k\Omega) = 1.5205V.$$

$$\therefore R_1 = 1.194 k\Omega$$

D. Find the power efficiency when $V_{out} = 10 \sin \omega t$?

Ignore the power dissipation at pre-driver and assume that each transistor carries a negligible current around $V_{out} = 0$ and turns off for half of the period.

$$\eta = \frac{P_{out}}{P_{out} + P_{av,n} + P_{av,p} + 2\cdot P_r}$$

$$P_{out} = \frac{1}{2}\cdot\frac{V_p^2}{R_L} = \frac{1}{2}\cdot\frac{100}{10} = 5\ W$$

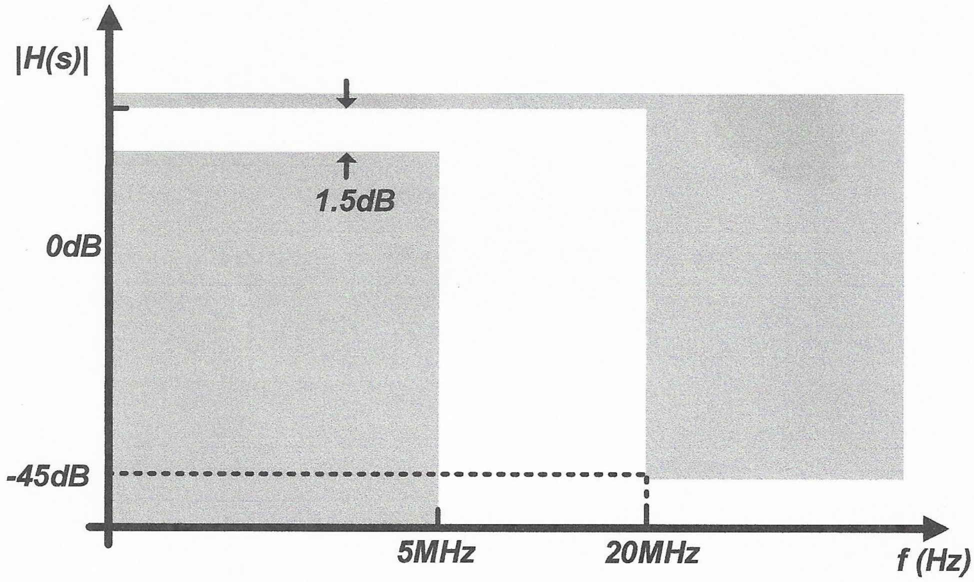
$$P_r = \frac{1}{T}\int_{\frac{T}{2}}^{T} I^2 r\, dt = \frac{1}{T}\int_{\frac{T}{2}}^{T} \left(\frac{10 \sin \omega t}{R_L}\right)^2 r\, dt = \frac{1}{4}\, 0.0875 = 0.021875\ W$$

$$P_{av,n} = \frac{1}{T}\int_{\frac{T}{2}}^{T} \left(V_{cc} - V_{out} - \frac{V_{out}}{R_L}\cdot r\right)\cdot\frac{V_{out}}{R_L}\, dt = \frac{10V_{cc}}{\pi R_L} - \frac{100}{4R_L}\left(1 + \frac{r}{R_L}\right)$$

$$= 10.847 W.$$

$$\therefore \eta = \frac{5}{10.847\times 2 + 5 + 0.021875\times 2} \Rightarrow 18.7\%$$

[1] Answer the following questions. [00 points]



Transfer function & Poles of the Butterworth Response
$\lvert H(jw)\rvert = \dfrac{1}{\sqrt{1+\left(\frac{\omega}{\omega_o}\right)^{2n}}}$
$p_k = \omega_o \exp\frac{j\pi}{2}\exp\left(j\frac{(2k-1)\pi}{2n}\right), k = 1,2,\ldots,n$
Transfer function & Poles of the Chebyshev Response
$\lvert H_{PB}(jw)\rvert = \dfrac{1}{\sqrt{1+\epsilon^2\cos^2\left(ncos^{-1}\left(\frac{\omega}{\omega_0}\right)\right)}}$ $\quad \lvert H_{SB}(jw)\rvert = \dfrac{1}{\sqrt{1+\epsilon^2\cosh^2\left(ncosh^{-1}\left(\frac{\omega}{\omega_0}\right)\right)}}$
$p_k = -\omega_0\sin\frac{(2k-1)\pi}{2n}\sinh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right) + j\omega_0\cos\frac{(2k-1)\pi}{2n}\cosh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right), k = 1,2,\ldots,n$
Useful Equations
$\sinh(x) = \frac{e^x - e^{-x}}{2}$, $\cosh(x) = \frac{e^x + e^{-x}}{2}$, $\sinh^{-1}(x) = \ln\left(x+\sqrt{x^2+1}\right)$, $\cosh^{-1}(x) = \ln\left(x+\sqrt{x^2-1}\right)$

A. Determine the minimum order of each filter (Butterworth Response & Chebyshev Response). Assume the bandwidth of Chebyshev filter is 5MHz. [00points]

< Butterworth >

$\cdot\ |H(j\omega)|_{\omega=2\pi\cdot 5M} \geq -1.5dB = 0.841$

$$\rightarrow \frac{1}{\sqrt{1+\left(\frac{2\pi\times 5M}{\omega_0}\right)^{2n}}} = 0.841 \quad \cdots ①$$

$\cdot\ |H(j\omega)|_{\omega=2\pi\cdot 20M} \leq -45dB = 5.6\times10^{-3}$

$$\rightarrow \frac{1}{\sqrt{1+\left(\frac{2\pi\cdot 20M}{\omega_0}\right)^{2n}}} = 5.6\times10^{-3} \quad \cdots ②$$

$\Rightarrow$ ①&② : $n \doteqdot 4.058$ $\Rightarrow$ minimum order = 5

< Chebyshev >

$\circ\ \omega_0 = 2\pi\cdot 5M.$

$\cdot\ |H(j\omega)|_{\omega=2\pi\cdot 5M} = -1.5dB = 0.841$

$$\rightarrow \frac{1}{\sqrt{1+\epsilon^2}} = 0.841 \Rightarrow \epsilon \doteqdot 0.643 \quad \cdots ③$$

$\cdot\ |H(j\omega)|_{\omega=2\pi\cdot 20M} = -45dB = 5.6\times10^{-3}$

$$\rightarrow \frac{1}{\sqrt{1+\epsilon^2\cosh^2\left(n\cdot\cosh^{-1}\frac{2\pi\cdot 20M}{\omega_0}\right)}} = 5.6\times10^{-3} \quad \cdots ④$$

③&④ $\Rightarrow$ $n \doteqdot 3.06$

$\Rightarrow$ minimum order = 4

B. Using determined value of n above, get the range of the optimal natural frequency (ω_o) of the Butterworth filter. [00points]

From A,

① : $$\frac{1}{\sqrt{1+\left(\frac{2\pi\times 5M}{\omega_0}\right)^{2n}}} > 0.841 \underset{(n=5)}{\Rightarrow} \left(\frac{5M}{f_0}\right)^{10} < \frac{1}{(0.841)^2} - 1 = 0.414 \quad \cdots ⑤$$

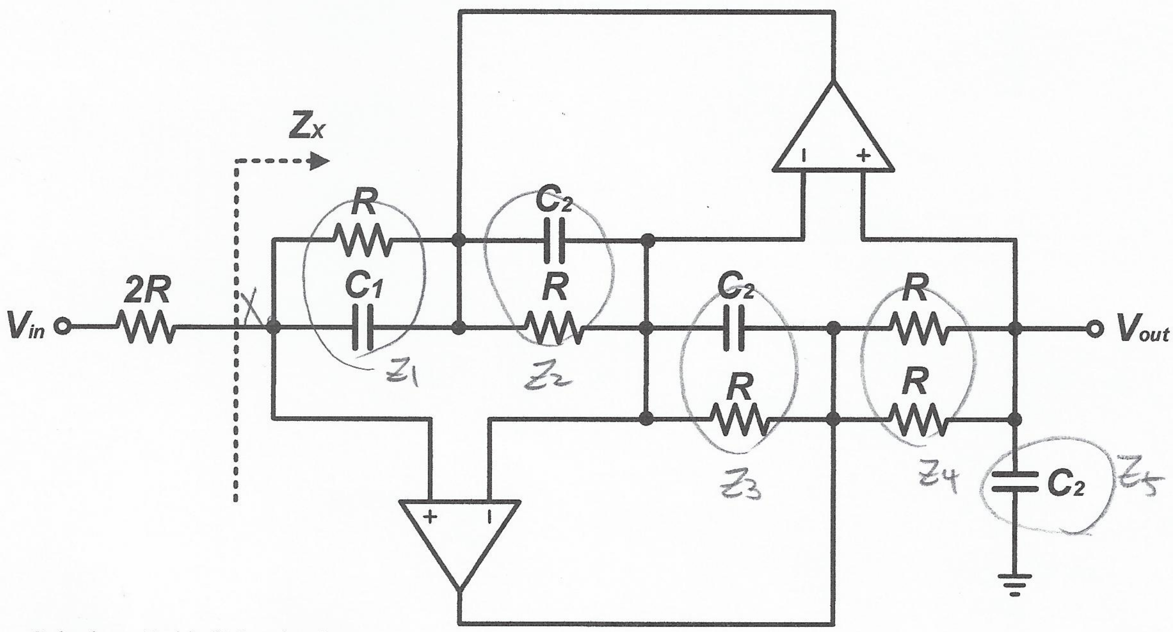
② : $$\frac{1}{\sqrt{1+\left(\frac{2\pi\times 20M}{\omega_0}\right)^{2n}}} < 5.6\times10^{-3} \underset{(n=5)}{\Rightarrow} \left(\frac{20M}{f_0}\right)^{10} > \frac{1}{(5.6\times10^{-3})^2} - 1 = 31886.7 \quad \cdots ⑥$$

$$⑤\&⑥ \Longrightarrow \frac{5M}{(0.414)^{\frac{1}{10}}} < f_0 < \frac{20M}{(31886.7)^{\frac{1}{10}}} \Rightarrow 5.46MHz < f_0 < 7.09MHz$$

$\rightarrow$ Optimum natural frequency $f_0 = \sqrt{(5.46M)(7.09M)} = 6.22MHz.$

$\rightarrow$ Optimum natural frequency $\omega_0 = 2\pi\cdot f_0 = 39.09 M rad/s$

[2] Answer the following questions. Assume all op-amps are ideal. [00 points]



A. Calculate $Z_X(s)$. [00 points]

$$Z_X(s) = \frac{Z_1 \cdot \cancel{Z_3}}{\cancel{Z_2} \cdot Z_4} \cdot Z_5 = \frac{R}{(RC_1 s+1)} \cdot \frac{1}{R/2} \cdot \frac{1}{C_2 s} = \frac{2}{C_2 s(RC_1 s+1)}$$

B. Derive the transfer function H(s). [00 points]

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{V_X}{V_{in}} \quad (\because V_X = V_{out})$$

$$= \frac{Z_X}{2R + Z_X} = \frac{\frac{2}{C_2 s(RC_1 s+1)}}{2R + \frac{2}{C_2 s(RC_1 s+1)}} = \frac{1}{R^2 C_1 C_2 s^2 + RC_2 s + 1} = \frac{\frac{1}{R^2 C_1 C_2}}{s^2 + \frac{1}{RC_1} s + \frac{1}{R^2 C_1 C_2}}$$

C. Determine the sensitivity of Q to a change in C1, S_{C1}^{Q}, and the sensitivity of ω_n to a change in R, $S_R^{\omega_n}$. [00 points]

From B, $w_n = \frac{1}{R\sqrt{C_1 C_2}} \longrightarrow S_R^{w_n} = \frac{dw_n}{dR} \cdot \frac{R}{w_n} = -1$

$$\frac{w_n}{Q} = \frac{1}{RC_1} \longrightarrow Q = \sqrt{\frac{C_1}{C_2}} \rightarrow S_{C_1}^{Q} = \frac{dQ}{dC_1} \cdot \frac{C_1}{Q} = \frac{1}{2}.$$

D. Design a Chebyshev filter of the problem 4 having high Q factor based on this filter structure, 'General Impedance Converter'. Every R is 1k ohm and you need to get the value of C_1, C_2.

From Problem 4, $\underline{\epsilon = 0.643,\ n=4}$ $\rightarrow$ $\sinh^{-1}\frac{1}{\epsilon} \doteqdot 1.225$

$$\begin{cases} \sinh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right) \doteqdot 0.311 \\ \cosh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right) \doteqdot 1.047 \end{cases}$$

$$P_{1,4} = -W_0 \cdot \sin\frac{\pi}{8} \cdot \sinh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right) \pm jW_0 \cos\frac{\pi}{8} \cosh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right)$$

$$= -0.119\,W_0 \pm j\,0.967\,W_0$$

$$P_{2,3} = -W_0 \sin\frac{3\pi}{8} \cdot \sinh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right) \pm jW_0 \cos\frac{3}{8}\pi \cosh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}\right)$$

$$= -0.287\,W_0 \pm j\,0.401\,W_0$$

Highest Q factor는 pole이 Imaginary axis에 가까울때.. $\Rightarrow$ P_1, P_4

$$H(s) = \frac{P_1 \cdot P_4}{(s-P_1)(s-P_4)} = \frac{0.949\,W_0^2}{s^2 + 0.238\,W_0 s + 0.949\,W_0^2}$$

- Comparing with the result of the B.

$$\left(H(s) = \frac{\frac{1}{R^2C_1C_2}}{s^2 + \frac{1}{RC_1}s + \frac{1}{R^2C_1C_2}} \right)$$

$$\Rightarrow \frac{1}{RC_1} = 0.238 \cdot W_0 \Rightarrow C_1 = \frac{1}{R} \cdot \frac{1}{0.238\,W_0} \doteqdot 133.7\text{ pF}$$

$$\frac{1}{R^2C_1C_2} = 0.949\,W_0^2 \Rightarrow C_2 = \frac{1}{R^2C_1} \cdot \frac{1}{0.949 \cdot W_0^2} \doteqdot 8\text{ pF}$$