School of Mech & Aero Eng
Seoul National University

Eng Probability
April 17, 2008

MIDTERM

- Do not open exam until told to do so.
- 'How you arrived at your answer' is much more important than the answer itself. Read the following problems carefully, and make sure you show your work *step by step*.
- You can attach extra pages if necessary. Please use a separate sheet for each problem.
- Ask questions if you don't understand what you are being asked, and **GOOD LUCK !**

Student ID: ________________

Name: ________________

1	/ 10
2	/ 15
3	/ 10
4	/ 15
5	/ 10
6	/ 15
7	/ 15
8	/ 10
Total	/ 100

1. [`5+5=10 pts`]

(a) You want to find someone with the same birthday as yours (out of 365 days per year). What is the least number of people you need to ask to have a 50 % chance of finding at least one matches?

(b) A girl and her friend are supposed to meet between 1 and 2 PM. Each comes at a random moment between 1 and 2 PM and waits for exactly 10 minutes. The meeting is successful only when the other person arrives within the 10-minute interval. What is the probability that the two people successfully meet?

2. [`15 pts`] A biased coin is tossed repeatedly. Each toss is independent with a probability p of a head. Show that the probability that there is a run of r heads in a row before there is a run of s tails is

$$\frac{p^{r-1}(1-q^s)}{p^{r-1}+q^{s-1}-p^{r-1}q^{s-1}} \ ,$$

where r and s are positive integers.

3. [`5+5=10 pts`] Let X and Y be independent random variables with common distribution function F and density function f.

(a) Compute the distribution function and density function of $V = \max(X, Y)$.

(b) Compute the distribution function and density function of $U = \min(X, Y)$.

4. [`15 pts`]Let X have the normal distribution $N(0,1)$.

(a) Compute the density function of $Y = e^X$.

(b) Let $Z = \sigma(\mu + X)$. Show that $E[(Z-\mu)g(Z)] = \sigma^2 E[g'(Z)]$

5. [`10 pts`] A point (X, Y) is chosen uniformly at random in the unit circle. find the joint density function of $R^2 = X^2 + Y^2$ and X.

6. [`15 pts`] A random number N of dice is thrown. Let $P(N = i) = 2^{-i},\ i \geq 1$, and S denote the sum of the scores. Find the probability that

(a) S=4 given N=even.

(b) the largest number shown by any die is less than or equal to m, where S is unknown.

(c) the largest number shown by any die is equal to m, where S is unknown.

7. [15 pts]
Let X and Y have joint density function

$$f(x, y) = \frac{1}{x}, \quad 0 \le y \le x \le 1.$$

(a) compute the density functions of X.

(b) compute the density function of $X + Y$.

8. [`10 pts`] If the density of X is given by

$$f(x) = \begin{cases} ax + bx^2 & 1 > x > 0 \\ 0 & else \end{cases} \quad ,$$

and $E[X] = 0.7$, compute $var(X)$ and $P(X > 0.9)$.