ID number: Heat Transfer

Name: 1st Semester, 2008

Problem	1	2	3	4	5	Total
Points						

Mid Term Exam 1

(April 14)

1. Consider a very long fin with uniform cross-sectional area shown below. The temperature distribution for the long fin is given by

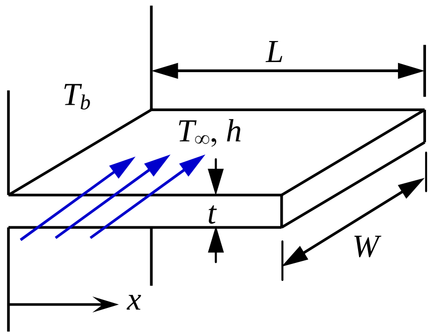
$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\exp\left(-\sqrt{\frac{hP}{kA_c}}x\right)$$

When T_∞ = 25°C, h = 100 W/m^2·K, T_b = 100°C, k = 400 W/m·K, L = 10 cm, W = 10 cm, t = 0.2 cm, find

a) fin effectiveness

b) fin resistance

c) fin efficiency



a) $\varepsilon_f=\dfrac{q_f}{q_{\text{w/o}}}$, $q_f=-kA_c\left.\dfrac{dT}{dx}\right|_{x=0}=kA_c\left(T_b-T_\infty\right)\sqrt{\dfrac{hP}{kA_c}}=\sqrt{hPkA_c}\left(T_b-T_\infty\right)$,

$$q_{\text{w/o}}=hA_c\left(T_b-T_\infty\right)$$

$$\varepsilon_f=\frac{q_f}{q_{\text{w/o}}}=\frac{\sqrt{hPkA_c}\left(T_b-T_\infty\right)}{hA_c\left(T_b-T_\infty\right)}=\sqrt{\frac{kP}{hA_c}}=\sqrt{\frac{2k\left(W+t\right)}{hWt}}=\sqrt{\frac{2\times 400\left(10+0.2\right)\times 10^{-2}}{100\times 10\times 0.2\times 10^{-4}}}=63.9$$

b) $q_f=\dfrac{\left(T_b-T_\infty\right)}{R_{t,f}}\rightarrow R_{t,f}=\dfrac{\left(T_b-T_\infty\right)}{q_f}=\dfrac{1}{\sqrt{hPkA_c}}$

$$R_{t,b}=\frac{\left(T_b-T_\infty\right)}{q_b}=\frac{\left(T_b-T_\infty\right)}{hA_c\theta_b}=\frac{1}{hA_c}$$

$$\varepsilon_f = \frac{R_{t,b}}{R_{t,f}} = \frac{\sqrt{hPkA_c}}{hA_c} = \sqrt{\frac{kP}{hA_c}} = 63.9$$

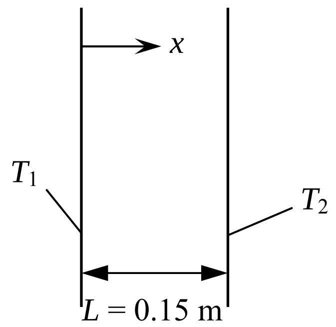
c) $$\eta_f = \frac{q_f}{q_{\max}} = \frac{\sqrt{hPkA_c}\left(T_b - T_\infty\right)}{hA_f\left(T_b - T_\infty\right)} = \frac{\sqrt{hPkWt}}{hPL} = \frac{1}{L}\sqrt{\frac{kWt}{hP}}$$

$$= \frac{1}{0.1}\sqrt{\frac{400\times 0.1\times 0.002}{100\times 2\left(0.1+0.002\right)}} = 0.626$$

2. Consider a slab of thickness L = 0.15 m as shown below. One side (x = 0) is at T_1 = 400 K and the other side (x = 0.15) at T_2 = 300 K. The thermal conductivity of the slab depends on temperature in such a way that $k(T) = k_0 e^{aT}$, where k_0 = 0.05 W/m·K, $a = 4.7\times10^{-3}$ K^{-1}.

a) Find the steady-state temperature distribution $T(x)$ in the slab. Hint: $\frac{d}{dx}\left(k(T)\frac{dT}{dx}\right)=0$

b) What is the heat flux q'' [W/m^2] across the slab?



a) $k(T)\frac{dT}{dx}=C_1$, $k_0 e^{aT}dT = C_1 dx$, $\frac{k_0}{a}d\left(e^{aT}\right)=C_1 dx$

$$e^{aT} = \frac{a}{k_0}C_1 x + C_2, \quad T(0)=T_1 \rightarrow e^{aT_1} = C_2$$

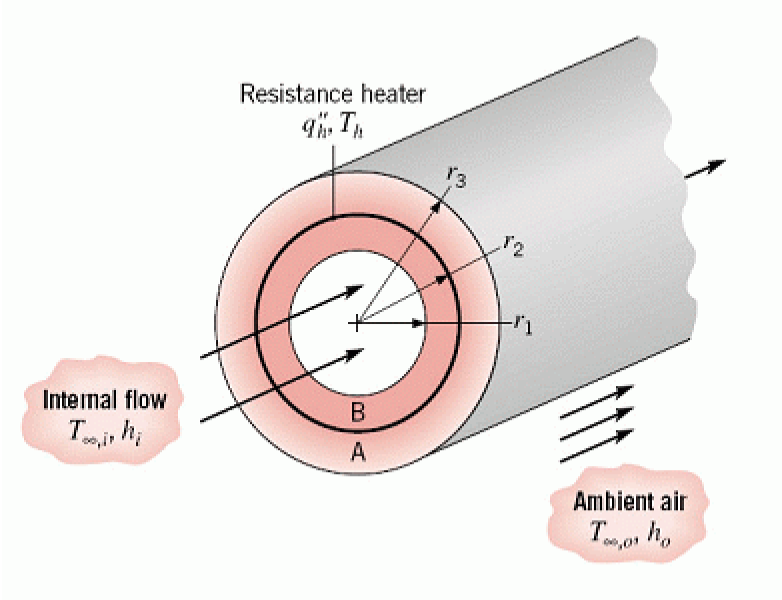
$$T(L)=T_2 \rightarrow e^{aT_2} = \frac{a}{k_0}C_1 L + e^{aT_1}, \quad C_1 = \frac{k_0\left(e^{aT_2}-e^{aT_1}\right)}{aL}$$

$$T(x)=\frac{1}{a}\ln\left(\frac{a}{k_0}C_1 x + C_2\right)=\frac{1}{a}\ln\left(\frac{a}{k_0}\frac{k_0\left(e^{aT_2}-e^{aT_1}\right)}{aL}x+e^{aT_1}\right)=\frac{1}{a}\ln\left(\left(e^{aT_2}-e^{aT_1}\right)\frac{x}{L}+e^{aT_1}\right)$$

b) $$q''=-k\frac{dT}{dx}=-k_0 e^{aT}\frac{dT}{dx}=-C_1=-\frac{k_0\left(e^{aT_2}-e^{aT_1}\right)}{aL}$$

$$=\frac{0.05\left(e^{4.7\times10^{-3}\times400}-e^{4.7\times10^{-3}\times300}\right)}{4.7\times10^{-3}\times0.15}=174.3\ \text{W/m}^2$$

3. A composite cylindrical wall is composed of two materials of thermal conductivity k_A and k_B, which are separated by a very thin, electric resistance heater for which interfacial contact resistances are negligible. Liquid pumped through the tube is at a temperature $T_{\infty,i}$ and provides a convection coefficient h_i at the inner surface of the composite. The outer surface is exposed to ambient air, which is at $T_{\infty,o}$ and provides a convection coefficient h_o. Under steady-state conditions, a uniform heat flux of q_h'' is dissipated by the heater.

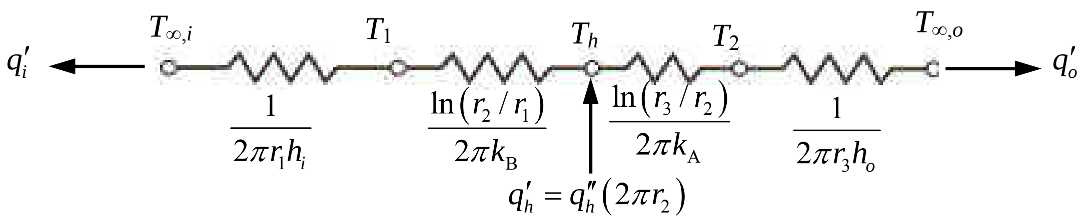


a) Sketch the equivalent thermal circuit of the system and express all resistances in terms of relevant variables. Hint: $R_{t,\text{cond}} = \dfrac{\ln(r_2/r_1)}{2\pi Lk}$, $R_{t,\text{conv}} = \dfrac{1}{2\pi r_1 L h_1}$

b) Obtain an expression that may be used to determine the heater temperature, T_h.

c) Obtain an expression for the ratio of heat flows to the outer and inner fluids, q_o'/q_i'. How might the variables of the problem be adjusted to minimize this ratio?

a)



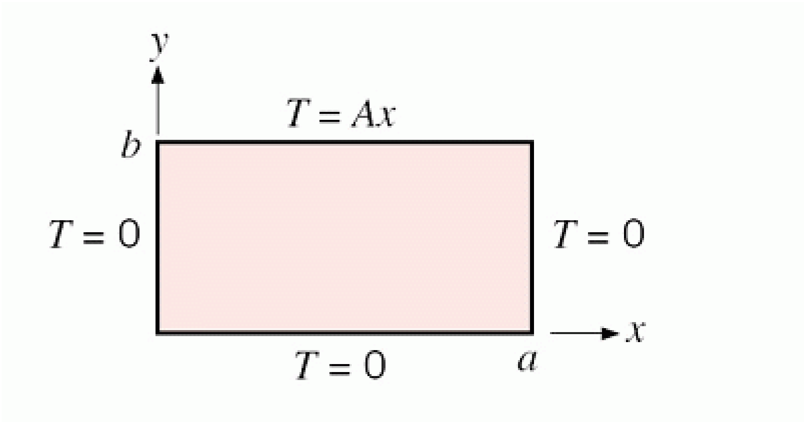
b) $$q_h' = q_h''(2\pi r_2) = q_i' + q_o' = \frac{T_h - T_{\infty,i}}{\dfrac{1}{2\pi r_1 h_i} + \dfrac{\ln(r_2/r_1)}{2\pi k_B}} + \frac{T_h - T_{\infty,o}}{\dfrac{1}{2\pi r_3 h_o} + \dfrac{\ln(r_3/r_2)}{2\pi k_A}}$$

c) $$\frac{q_o'}{q_i'}=\frac{\dfrac{T_h-T_{\infty,o}}{\dfrac{1}{2\pi r_3 h_o}+\dfrac{\ln\left(r_3/r_2\right)}{2\pi k}}}{\dfrac{T_h-T_{\infty,i}}{\dfrac{1}{2\pi r_1 h_i}+\dfrac{\ln\left(r_2/r_1\right)}{2\pi k}}}=\frac{\left(T_h-T_{\infty,o}\right)\left(\dfrac{1}{2\pi r_1 h_i}+\dfrac{\ln\left(r_2/r_1\right)}{2\pi k}\right)}{\left(T_h-T_{\infty,i}\right)\left(\dfrac{1}{2\pi r_3 h_o}+\dfrac{\ln\left(r_3/r_2\right)}{2\pi k}\right)}$$

To reduce q_o'/q_i', one could increase k_B, h_i, and r_3/r_2, while reducing k_A, h_o, and r_2/r_1.

4. A two dimensional rectangular plate is subjected to the boundary conditions shown. Derive an expression for the steady-state temperature distribution *T*(*x*,*y*).

Hint: $\int x\sin ax dx = \frac{\sin ax}{a^2} - \frac{x\cos ax}{a}$, $\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$



$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$ b.c. x: $T(0,y)=0,\ T(a,y)=0$ y: $T(x,0)=0,\ T(x,b)=Ax$

Assume $T(x,y) = X(x)Y(y)$, then $\frac{X''}{X} = -\frac{Y''}{Y} = -\lambda^2$

X: $X'' + \lambda^2 X = 0,\ X(0)=0,\ X(a)=0$

$X(x) = C_1 \sin\lambda x + C_2\cos\lambda x$, $x(0)=0 \to C_2 = 0$, $x(a)=0 \to C_1\sin\lambda a = 0$

To be non-trivial, $\lambda a = n\pi \to \lambda_n = \frac{n\pi}{a}$. Thus $X_n(x) = a_n \sin\frac{n\pi x}{a}$

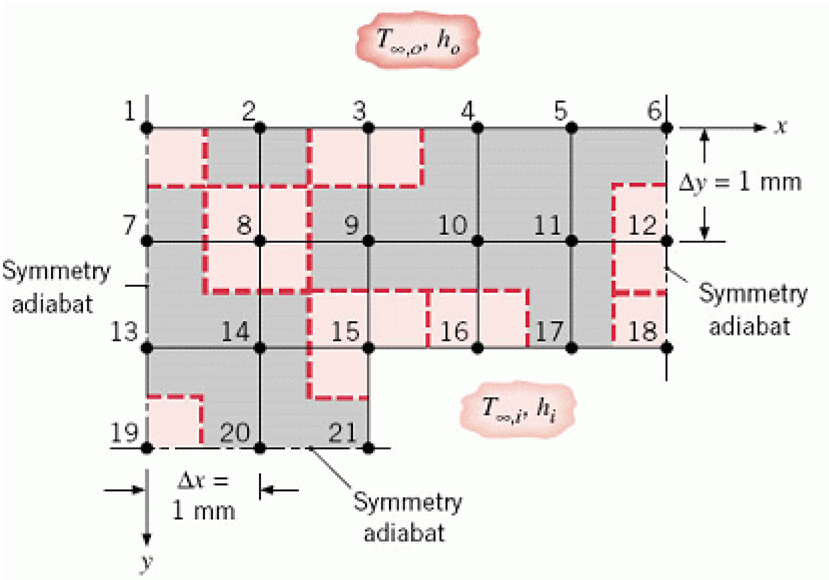
Y: $Y(y) = C_3\sinh\lambda y + C_4\cosh\lambda y$, $Y(0)=0$

$Y(0)=0 \to C_4 = 0$ Thus $Y_n(x) = b_n \sinh\frac{n\pi y}{a}$

$T(x,y) = \sum_{n=1}^{\infty} c_n \sin\frac{n\pi x}{a}\sinh\frac{n\pi y}{a}$, $T(x,b) = Ax = \sum_{n=1}^{\infty} c_n \sin\frac{n\pi x}{a}\sinh\frac{n\pi b}{a}$

$$c_n = \frac{\int_0^a Ax\sin\frac{n\pi x}{a}dx}{\sinh\frac{n\pi b}{a}\int_0^a \sin\frac{n\pi x}{a}dx}$$

5. The finite difference method is used to determine the temperature distribution in the solid shown below and the heat transfer rate from the hot gas flow to the coolant through the solid. The thermal conductivity of the solid is $k = 25$ W/m·K, and other conditions are; h_o = 1000 W/m^2·K, $T_{\infty,o}$ =1700 K, h_i = 200 W/mK, and $T_{\infty,i}$ = 400 K.



a) Write down the finite difference equations for nodal points 1, 3, 8, 12, 15, 16, 18, and 19 in terms the temperatures of neighboring nodes

b) Express the heat transfer rate per unit length q' to the channel in terms of the nodal temperatures.