ID number: Heat Transfer

Name: $1^{st}$ Semester, 2008

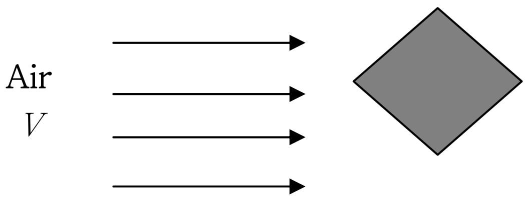
| Problem | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| Points | | | | | | |

**Final Exam**

(June 11)

1. Experimental measurements of the convection heat transfer coefficient for a square bar in cross flow yielded the following values (length of one side $L$ = 0.5 m):

$\bar{h}_1$ = 50 W/m$^2$·K when $V_1$ = 20 m/s, $\bar{h}_2$ = 40 W/m$^2$·K when $V_2$ = 15 m/s




Assume that the functional form of the Nusselt number is $\overline{\text{Nu}} = C\,\text{Re}^m\,\text{Pr}^n$, where $C$, $m$, $n$ are constants (5 pts each, 20 pts in total).

(a) What is the value of $m$?

(b) What will be the convection heat transfer coefficient for a similar bar with $L$ = 1 m when $V$ = 15 m/s?

(c) What will be the convection heat transfer coefficient for a similar bar with $L$ = 1 m when $V$ = 30 m/s?

(d) Would your results be the same if the diagonal of the bar, rather than its length of one side, were used as the characteristic length?

Solution

(a)

$$\frac{\bar{h}L}{k} = C\left(\frac{VL}{\nu}\right)^m \text{Pr}^n$$

$$\frac{\bar{h}_1}{\bar{h}_2} = \left(\frac{V_1}{V_2}\right)^m \rightarrow \frac{50}{40} = \left(\frac{20}{15}\right)^m \rightarrow m = \frac{\ln(5/4)}{\ln(4/3)} = 0.776$$

(b)

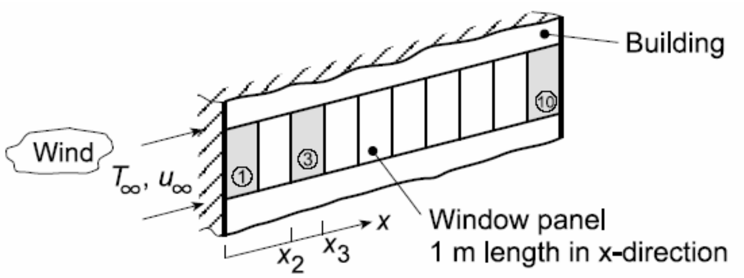
$$\frac{\bar{h}_3 L_3}{\bar{h}_1 L_1} = \left(\frac{V_3 L_3}{V_1 L_1}\right)^m \rightarrow \frac{\bar{h}_3}{50 \times 0.5} = \left(\frac{15}{20 \times 0.5}\right)^{0.776} \rightarrow \bar{h}_3 = 34.2\ \text{W/m}^2 \cdot \text{K}$$

(c)

$$\frac{\bar{h}_4 L_4}{\bar{h}_1 L_1} = \left(\frac{V_4 L_4}{V_1 L_1}\right)^m \rightarrow \frac{\bar{h}_4}{50 \times 0.5} = \left(\frac{30}{20 \times 0.5}\right)^{0.776} \rightarrow \bar{h}_4 = 58.6\ \text{W/m}^2 \cdot \text{K}$$

(d) If the diagonal of the bar is chosen as the characteristic length, $C$ would change while $m$ and $n$ remain the same.

2. Consider weather conditions for which the prevailing wind blows past the penthouse tower on a tall building. The tower length in the wind direction is 10 m and there are 10 window panels. For simplicity, it is assumed that material properties of air at a film temperature of 300 K can be used: $\nu = 16 \times 10^{-6}$ m²/s, $k = 26 \times 10^{-3}$ W/m·K, Pr = 0.7. The critical Reynolds number for turbulent transition is $Re_{x,c} = 5 \times 10^5$. (20 pts)




(a) Predict at which window the air flow transits from laminar to turbulent when the wind speed is $u_\infty = 5$ m/s.

(b) Calculate the average convection coefficient for the first and third window panels.

Solution

(a) $\mathrm{Re}_{x,c} = \dfrac{u_\infty x}{\nu} = \dfrac{5x}{16\times10^{-6}} = 5\times10^5 \rightarrow x = 1.6 \text{ m}$

Thus, transition occurs at the second window.

(b)

1) first window: laminar $\quad \overline{\mathrm{Nu}}_x = 0.664\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$

$$\bar{h}_1 = 0.664\frac{k}{x_1}\left(\frac{u_\infty x_1}{\nu}\right)^{1/2}\mathrm{Pr}^{1/3} = 0.664\times\frac{26\times10^{-3}}{1}\left(\frac{5\times1}{16\times10^{-6}}\right)^{1/2}(0.7)^{1/3} = 8.57\text{ W/m}^2\cdot\text{K}$$

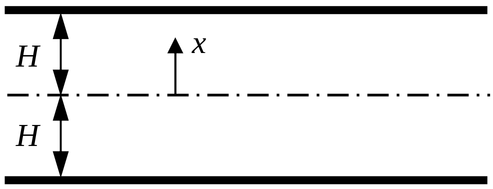
2) third window: turbulent, local Nusselt number: $\mathrm{Nu}_x = 0.0296\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3}$

$$h_3 = 0.0296\frac{k}{x}\left(\frac{u_\infty x}{\nu}\right)^{4/5}\mathrm{Pr}^{1/3} = 0.0296k\left(\frac{u_\infty}{\nu}\right)^{4/5}\mathrm{Pr}^{1/3}\,x^{-1/5}$$

$$\bar{h}_3 = 0.0296k\left(\frac{u_\infty}{\nu}\right)^{4/5}\mathrm{Pr}^{1/3}\int_2^3 x^{-1/5}dx$$

$$= 0.0296\times26\times10^{-3}\times\left(\frac{5}{16\times10^{-6}}\right)^{4/5}(0.7)^{1/3}\frac{5}{4}\left(3^{0.8}-2^{0.8}\right) = 14.2\text{ W/m}^2\cdot\text{K}$$

3. For a liquid metal flow between two parallel plates (two-dimensional channel) as shown below, the velocity and temperature profiles at a particular axial location may be approximated as being uniform and parabolic, respectively. That is, $u(x) = C_1$ and $T(x) - T_s = C_2\left\{1 - (x/H)^2\right\}$, where $C_1$ and $C_2$ are constants and $H$ is the half height of the channel. Determine the corresponding value of the mean (or bulk) temperature $T_m$ and the Nusselt number $Nu_H$ (= $hH/k$) at this axial position. (20 pts)




Solution

$$\rho c_p H u T_m = \int_0^H \rho c_p u T dx$$

$$\rightarrow T_m = \frac{1}{H}\int_0^H \left[T_s + C_2\left\{1 - \left(\frac{x}{H}\right)^2\right\}\right]dx = \frac{1}{H}\left[T_s x + C_2\left(x - \frac{1}{3}\frac{x^3}{H^2}\right)\right]_0^H = T_s + \frac{2}{3}C_2$$

$$q'' = k\left.\frac{dT}{dx}\right)_{x=H} = h\left(T_s - T_m\right),\quad \frac{dT}{dx} = -2C_2\frac{x}{H^2},\quad k\left.\frac{dT}{dx}\right)_{x=H} = -\frac{2kC_2}{H}$$

$$-\frac{2kC_2}{H} = h\left(T_s - T_s - \frac{2}{3}C_2\right) = -\frac{2}{3}C_2 h \rightarrow h = 3\frac{k}{H} \rightarrow \mathrm{Nu}_H = \frac{hH}{k} = 3$$

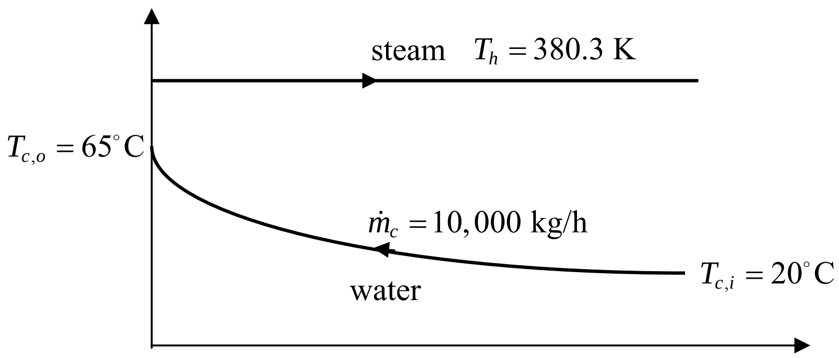
4. The feedwater heater for a boiler supplies 10,000 kg/h of water at 65°C. The feedwater has an inlet temperature of 20°C and is to be heated in a single-shell, two-tube pass heat exchanger by condensing steam at 1.30 bars. The overall heat transfer coefficient is 2000 $W/m^2 \cdot K$. Using both the LMTD and NTU methods, determine the required heat transfer area. What is the steam condensation rate? Assume correction factor $F = 1$.

Properties: Steam (1.3 bar, saturated): $T_h = 380.3$ K, $h_{fg} = 2238 \times 10^3$ J/kg·K; Water ($T$ = 316 K): $c_p = 4179$ J/kg·K, $\mu = 725 \times 10^{-6}$ $N\cdot s/m^2$, $k = 0.625$ W/m·K, Pr = 4.85.
Note that some properties are necessary, while some are not.

For single-shell and two tube passes,

$$\text{NTU} = -\left(1+C_r^2\right)^{-1/2} \ln\left(\frac{E-1}{E+1}\right), \quad E = \frac{2/\varepsilon - \left(1+C_r\right)}{\left(1+C_r^2\right)^{1/2}} \quad (C_r = C_{\min}/C_{\max})$$

Solution




1) LMTD method

$$q = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right) = UA\Delta T_{\text{lm}}$$

$$A = \frac{\dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)}{U\Delta T_{\text{lm}}} = \frac{\dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)\ln\dfrac{T_{h,i} - T_{c,o}}{T_{h,o} - T_{c,i}}}{U\left[\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)\right]}$$

$$= \frac{10000 \times 4179 \times \left(65-20\right)\ln\dfrac{107-65}{107-20}}{3600 \times 2000 \times \left[\left(107-65\right) - \left(107-20\right)\right]} = 4.23\ \text{m}^2$$

2) $\varepsilon$-NTU method

$$\text{NTU} = \frac{UA}{C_{\min}} \rightarrow A = \frac{\text{NTU} \times C_{\min}}{U}$$

$$\varepsilon = \frac{q}{q_{\max}} = \frac{\dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)}{C_{\min}\left(T_{h,i} - T_{c,i}\right)} = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{65-20}{107-20} = 0.517$$

$$C_r = \frac{C_{\min}}{C_{\max}} = 0$$

$$E = \frac{2/\varepsilon - \left(1 + C_r\right)}{\left(1 + C_r^2\right)^{1/2}} = \frac{2}{0.517} - 1 = 2.87$$

$$\text{NTU} = -\left(1 + C_r^2\right)^{-1/2} \ln\left(\frac{E-1}{E+1}\right) = -\ln\frac{2.87-1}{2.87+1} = 0.727$$

$$A = \frac{\text{NTU} \times C_{\min}}{U} = \frac{0.727 \times 10000 \times 4179}{2000 \times 3600} = 4.22\ \text{m}^2$$

3) steam condensation rate

$$q = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right) = \dot{m}_h h_{fg}$$

$$\dot{m}_h = \frac{\dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)}{h_{fg}} = \frac{10000 \times 4179 \times \left(65 - 20\right)}{2238 \times 10^3} = 840\ \text{kg/h}$$

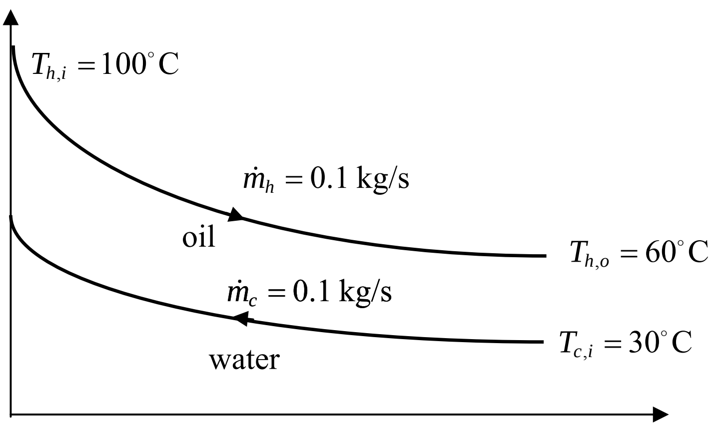
5. A concentric tube heat exchanger for cooling lubricating oil is comprised of a thin-walled inner tube of 20-mm diameter carrying water and an outer tube of 40-mm diameter carrying the oil. The exchanger operates in counterflow with an overall heat transfer coefficient of 50 W/m$^2$·K. The inlet temperatures are 100°C and 30°C for oil and water, respectively and the flow rates are the same at 0.1 kg/s for both fluids. The material properties are given below (20 pts)

| **Properties** | **Water** | **Oil** |
|---|---|---|
| $\rho$ (kg/m$^3$) | 1000 | 800 |
| $c_p$ (J/kg·K) | 4200 | 1900 |
| $\nu$ (m$^2$/s) | $7\times 10^{-7}$ | $1 \times 10^{-5}$ |
| $k$ (W/m·K) | 0.64 | 0.134 |
| Pr | 4.7 | 140 |

(a) If the outlet temperature of the oil is 60°C, determine the total heat transfer rate and the outlet temperature of the water.

(b) Determine the length ($L$) required for the heat exchanger.

Solution




(a) $q = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right) = \dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right) = 0.1\times 1900\times\left(100-60\right) = 7600 \text{ W}$

$$T_{c,o} = T_{c,i} + \frac{\dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right)}{\dot{m}_c c_{p,c}} = 30 + \frac{1900\times 40}{4200} = 48.1^\circ\text{C}$$

(b) $$A = \pi D_i L = \frac{\dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)}{U\Delta T_{\text{lm}}},\quad L = \frac{\dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)}{\pi D_i U \Delta T_{\text{lm}}} = \frac{\dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)\ln\dfrac{T_{h,i} - T_{c,o}}{T_{h,o} - T_{c,i}}}{\pi D_i U\left[\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)\right]}$$

$$= \frac{0.1\times 4200\times\left(48.1-30\right)\ln\left[\left(100-48.1\right)/\left(60-30\right)\right]}{\pi\times 20\times 10^{-3}\times 50\times\left[\left(100-48.1\right)-\left(60-30\right)\right]} = 60.6 \text{ m}$$

Table. Summary of convection heat transfer correlations for external flow.

| **Correlation** | **Geometry** | **Condition** |
|---|---|---|
| $\mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$ | Flat plate | Laminar, local, $T_f$, $0.6 \le \mathrm{Pr}$ |
| $\overline{\mathrm{Nu}}_x = 0.664\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$ | Flat plate | Laminar, average, $T_f$, $0.6 \le \mathrm{Pr}$ |
| $\mathrm{Nu}_x = 0.0296\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3}$ | Flat plate | Turbulent, local, $T_f$, $\mathrm{Re} \le 10^8$<br>$0.6 \le \mathrm{Pr} \le 60$ |
| $\overline{\mathrm{Nu}}_L = (0.037\,\mathrm{Re}_L^{4/5} - 871)\,\mathrm{Pr}^{1/3}$ | Flat plate | Mixed, average, $T_f$, $\mathrm{Re} \le 10^8$<br>$0.6 \le \mathrm{Pr} \le 60$ |