Mid-term Exam, Spring Semester, 2008 – Data Networks

Professor: Sunghyun Choi

2008-05-06 (Tue), 12:30pm-2:15pm

1. (10 points) We are given a bit transmission facility that transmits each bit correctly with probability $1-p$ and incorrectly with probability p. The bit errors are independent. We send bits in groups of 7 and we add a parity bit so that the groups of 8 bits that we send always contain an even number of 1's. What is the probability that a group of 8 bits arrive at the receiver with transmission errors that com not be detected?

(10 points)

If even number of error occurs, it cannot be detected in this case.

P_n means the probability which n number of error occurs in the data bits.

P_2: ${}_8C_2(1-p)^6p^2$

P_4: ${}_8C_4(1-p)^4p^4$

P_6: ${}_8C_6(1-p)^2p^6$

P_8: ${}_8C_8p^8$

Total probability: $P_2 + P_4 + P_6 + P_8$

$= {}_8C_2(1-p)^6p^2 + {}_8C_4(1-p)^4p^4 + {}_8C_6(1-p)^2p^6 + {}_8C_8p^8$

2. (20 points) Consider a stop-and-wait system with two-way traffic between nodes A and B. Assume that data frames are all of the some length and require D seconds for transmission. Acknowledgement frames require R seconds for transmission, and there is a propagation delay P on the link. Assume that A and B both have an unending sequence of packets to send, assume that no transmission errors occur, and assume that the time-out interval at which a node resends a previously transmitted packet is very large. Finally, assume that each sends new data packets and acknowledgements as fast as possible, subject of the rules of the stop-and-wait protocol. The rate at which packets are transmitted in each direction is $(D+R+2P)^{-1}$. Show that this is true when the starting time between A and B is

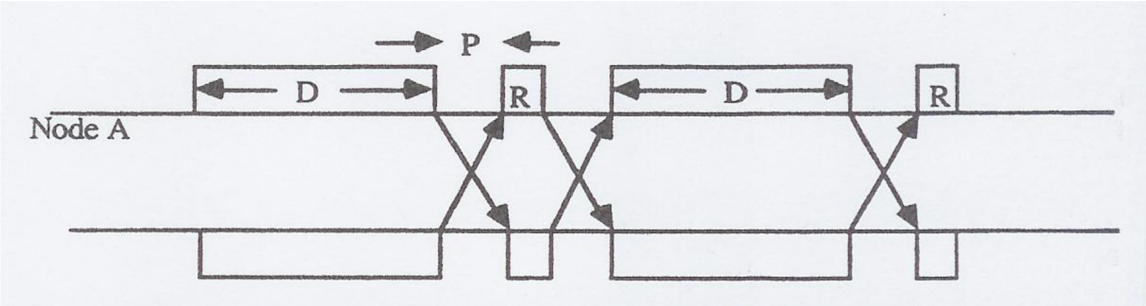
(a) synchronized (10 points)

(b) not synchronized (10 points)

Solution1: assuming two-way independent link

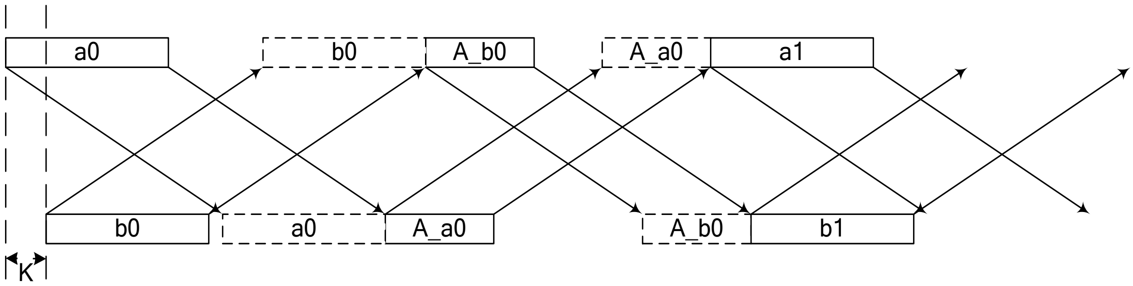
(a) synchronized (10 points)

Period of one data frame: D + P + R + P → true



(b) not synchronized (10 points)

Period of one data frame except first frame: D + P + R + P → true



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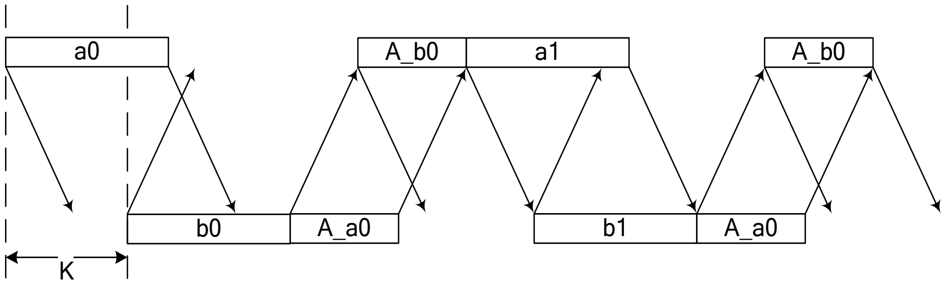
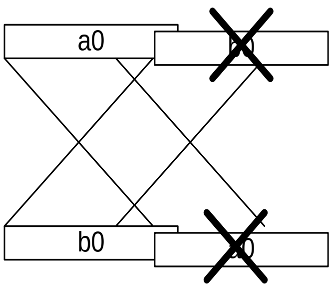


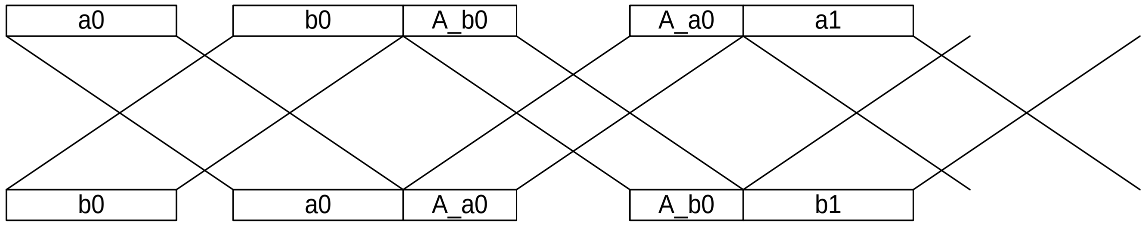
Solution2: assuming two-way shared link

(a) synchronized (10 points)

If P<D → false



If P>=D → true

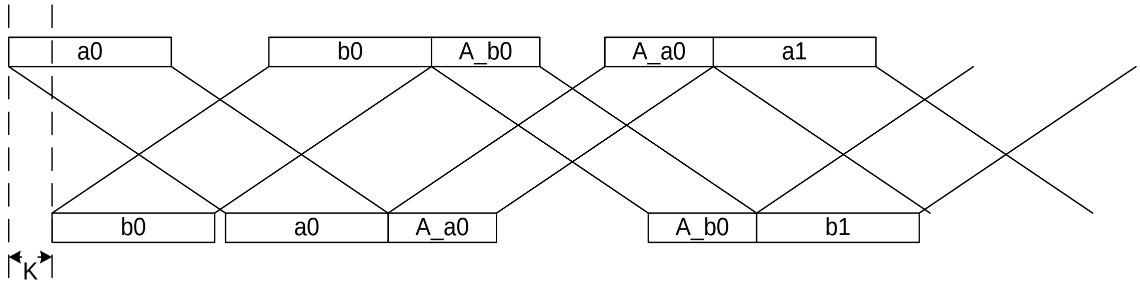


(b) not synchronized (10 points)

Define K as the difference of transmission start time between A and B.

If P<D→ false

If P>=D and |K| <= P-D → true



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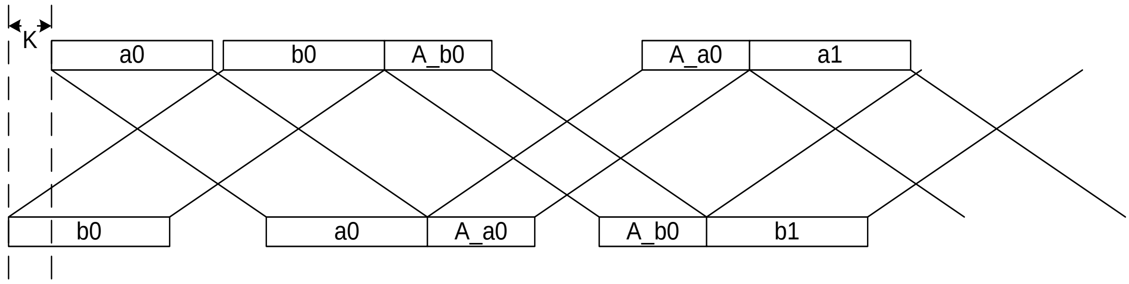
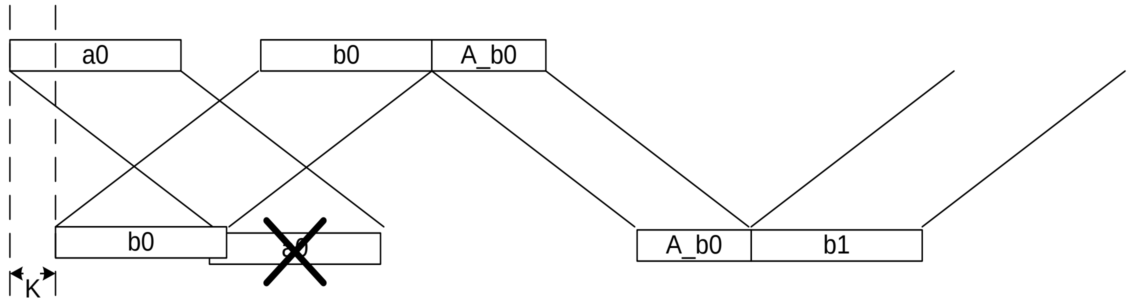


If P>=D and |K| > P-D → partially true (K<0 → true for A, K>0 → true for B)



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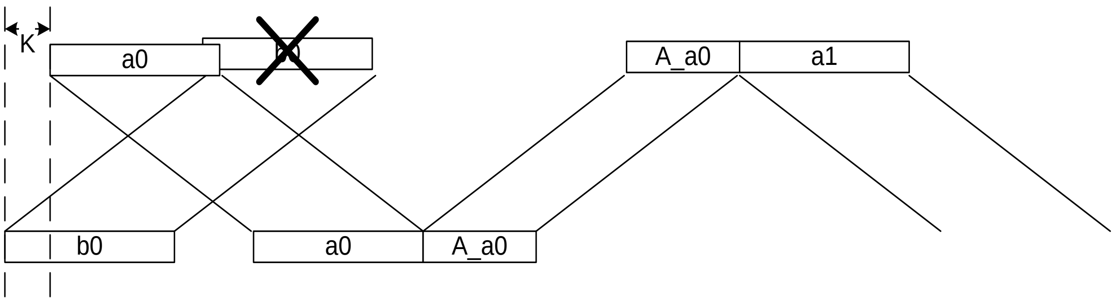


3. (10 points) Find an example of a pattern of ten errors that can not be detected by the use of horizontal and vertical parity checks.

(10 points)

If all the column and row has even number of errors, it cannot be detected.

WLOG, all the data bit can be assumed 1. Then, all the error bit is 0.

One example:

```
1 1 1 1 1 | 1          0 0 1 1 1 | 1
1 1 1 1 1 | 1          0 1 0 1 1 | 1
1 1 1 1 1 | 1    →     1 0 0 1 1 | 1
1 1 1 1 1 | 1          1 1 1 0 0 | 1
1 1 1 1 1 | 1          1 1 1 0 0 | 1
_______          _______
1 1 1 1 1        1 1 1 1 1
```

4. (35 points) A supermarket has two exponential checkout counters, each operating at rate μ. Arrivals are Poisson at rate λ. The counters operate in the following way;

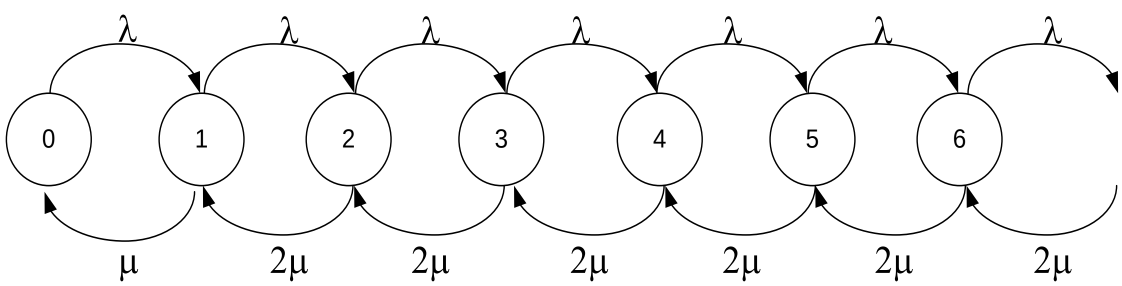
(i) one queue feeds both counters;

(ii) one counter is operated by a permanent checker and the other by a stock clerk who instantaneously begins checking whenever there are two or more customers in the system. The clerk returns to stocking whenever he completes a service, and there are less than two customers in the system.

(a) In steady state, compute, for all $n \geq 0$,

$\pi_n \triangleq$ Prob(there are n customers in the system). (15 point)

If we define the number of customers in the system as a state, we can draw the Markov chain as follows.



We assume that $\lambda/(2\mu)<1$.

It can be modeled as a M/M/2 queuing system, and hence, the stationary distribution is as follows.

$\pi_0\lambda=\pi_1\mu$ → $\pi_1=(\lambda/\mu)\pi_0$

$\pi_1\lambda=2\pi_2\mu$ → $\pi_2=(1/2)(\lambda/\mu)\pi_1$ → $\pi_2=2(1/2)^2(\lambda/\mu)^2\pi_0$

$\pi_{n-1}\lambda=2\pi_n\mu$ → $\pi_n=(1/2)(\lambda/\mu)\pi_{n-1}$ → $\pi_n=2(1/2)^n(\lambda/\mu)^n\pi_0$, n>2 (5 points)

and

$\Sigma\pi_n=1$ → $\pi_0+\pi_1+\pi_2+\pi_3+\ldots=1$

Using above equations, we can calculate π_0.

$\pi_0=(2\mu-\lambda)/(2\mu+\lambda)$

Then,

$\pi_0=(2\mu-\lambda)/(2\mu+\lambda)$

$\pi_1=(2\mu-\lambda)(\lambda/\mu)/(2\mu+\lambda)$

$\pi_n=2(1/2)^n(\lambda/\mu)^n(2\mu-\lambda)/(2\mu+\lambda)$ n>1

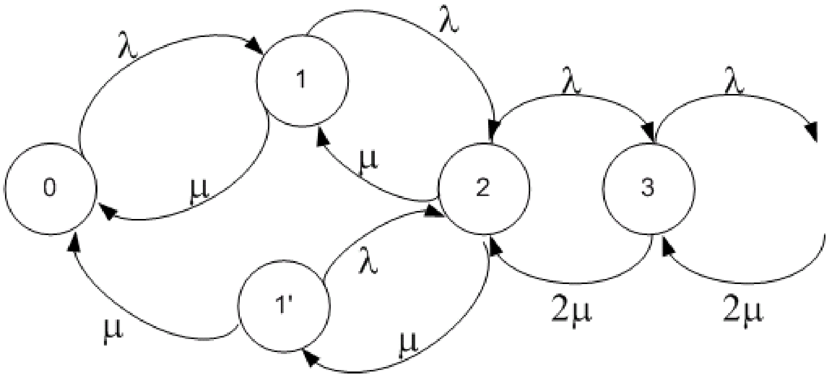
(Markov chain model : 5 points, solution: 5 points)

(b) In steady-state, compute

Prob (stock clerk is checking). (20 points)

---------Modified solution----------------

In order to calculate the probability, we need to divide the π_1 state into two states, π_1 and π_1'.



$\pi_0\lambda=\pi_1\mu+\pi_1'\mu$

$\pi_1\lambda+\pi_1\mu=\pi_0\lambda+\pi_2\mu$

$\pi_1'\mu+\pi_1'\lambda=\pi_2\mu$

Other equations are same as described in the previous solution.

$\pi_1=\lambda(2\mu-\lambda)/(2\mu+\lambda)/\mu-(1/2)(\lambda^2/\mu)(2\mu-\lambda)/(2\mu+\lambda)/(\mu+\lambda)$

$\pi_1'=(1/2)(\lambda^2/\mu)(2\mu-\lambda)/(2\mu+\lambda)/(\mu+\lambda)$ (10 points)

Others state probabilities are not changed.

Prob(stock clerk is checking): $1-\pi_0-\pi_1$

$=1-(2\mu-\lambda)/(2\mu+\lambda)-(\lambda)(2\mu-\lambda)/(2\mu+\lambda)/(\mu+\lambda)-(1/2)(\lambda^2/\mu)(2\mu-\lambda)/(2\mu+\lambda)/(\mu+\lambda)$ (10 points)

5. (25 points) Consider an M/G/1 system with $\lambda \bar{x} < 1$, where λ is the rate of the Poisson process describing the arrivals to the system and $\bar{x}$ is the mean service time. Suppose service begins when there are n customers in the system.

(a) Compute the expected time until there are $n-1$ customers in the system. (15 points)

Mean service time for one customer: $\mathring{X}$

Mean number of new arrived customers during $\mathring{X}$ time: $\lambda \cdot \mathring{X}$

Mean service time for $\lambda \cdot \mathring{X}$ customer: $(\lambda \cdot \mathring{X})\ \mathring{X}$

Mean number of new arrived customers during $(\lambda \cdot \mathring{X})\ \mathring{X}$ time: $\lambda \cdot (\lambda \cdot \mathring{X})\ \mathring{X}$

…

➔ $\mathring{X} + (\lambda \cdot \mathring{X})\ \mathring{X} + (\lambda \cdot (\lambda \cdot \mathring{X})\ \mathring{X})\ \mathring{X} + (\lambda \cdot (\lambda \cdot (\lambda \cdot \mathring{X})\ \mathring{X})\ \mathring{X})\ \mathring{X} + \ldots = \mathring{X}/(1 - \lambda \mathring{X})$

(b) Compute the expected time until the system is empty. (10 points)

$n\ \mathring{X}/(1 - \lambda \mathring{X})$

Final Exam, Spring Semester, 2008 – Data Networks

Professor: Sunghyun Choi

2008-06-13 (Fri), 11am-1pm

1. (20 points) Consider a slotted Aloha with an infinite set of nodes and with "perfect capture." That is, if more than one packet are transmitted in a slot, the receiver "locks onto" one of the transmissions and receives it correctly; feedback immediately informs each transmitting node about which node was successful and the unsuccessful packets are retransmitted in the next slot. Find the expected system delay assuming Poisson arrivals with overall rate $\lambda\ (\lambda < 1)$.

Sol1:

When there are n_i backlogged nodes upon arrival of i-th packet,

$$W_i = R_i + \sum_{j=1}^{n_i} t_j + y_i$$ (5points)

W_i : delay from the arrival of the ith packet until the beginning of the ith successful transmission,

R_i : residual time to the beginning of the next slot,

t_j : interval from the end of the (j-1)th subsequent success to the end of the jth subsequent success,

y_i : remaining interval until the beginning of the next successful transmission,

Due to the perfect capture, $\mathrm{E}[t_j] = 1,\ \mathrm{E}[y_i] = 0.$ (5points)

$\mathrm{E}[R_i] = \frac{1}{2},\ \mathrm{E}[n_i] = \lambda W$ (due to Little's theorem) (3points)

$$\therefore W = \frac{1}{2} + \lambda W,\ \Rightarrow W = \frac{1}{2(1-\lambda)}$$ (5points)

Expected system delay T = W + 1 slottime = $\frac{1}{2(1-\lambda)} + 1 = \frac{3-2\lambda}{2-2\lambda}$ (2points)

Sol2:

Using P-K formula for M/G/1 queue with vacations

$$W = \frac{\lambda E[X^2]}{2(1-\rho)} + \frac{E[V^2]}{2E[V]},$$ (5points)

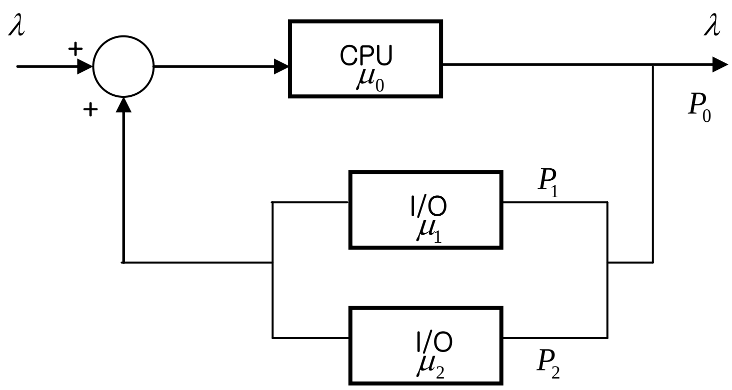
$E[X^2] = E[V^2] = E[V] = 1$ (due to perfect capture) (8points)

$\rho = \lambda / 1 = \lambda$

$$\therefore W = \frac{\lambda}{2(1-\lambda)} + \frac{1}{2}$$ (5points)

Expected system delay T = W + 1 $= \frac{\lambda}{2(1-\lambda)} + \frac{3}{2} = \frac{3-2\lambda}{2-2\lambda}$ (2points)

2. (25 points) Consider the system of the figure below. A computer CPU is connected to two I/O devices. Jobs enter the system according to a Poisson process with rate λ, use the CPU, and with probability P_i $(i=1,2)$, are routed to the i-th I/O device while with probability P_0 they exit the system. The service time of a job at the CPU (or the i-th I/O device, i=1,2) is exponentially distributed with mean $1/\mu_0$ (or $1/\mu_i$, $i=1,2$, respectively). We assume that all job service times at all queues are independent (including the times of successive visits to the CPU and the I/O devices of the same job).



(a) Determine the steady-state occupancy distribution of the system. (13 points)

(b) Construct an "equivalent" system with three queues in tandem that has the same steady-state occupancy distribution. (12 points)

Sol:

(a)

Let λ_0 be the arrival rate at the CPU and let λ_i be the arrival rate at I/O unit i.

$$\lambda_0 = \lambda + (1-P_0)\lambda_0 = \frac{\lambda}{P_0},$$

$$\lambda_1 = P_1\lambda_0 = \frac{P_1\lambda}{P_0}, \lambda_2 = P_2\lambda_0 = \frac{P_2\lambda}{P_0} \qquad \text{(3points)}$$

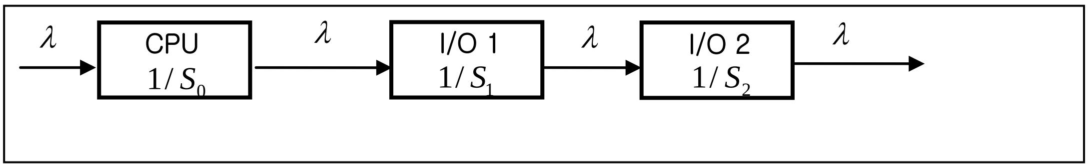
$$\rho_0 = \frac{\lambda_0}{\mu_0} = \frac{\lambda}{P_0\mu_0},$$

$$\rho_1 = \frac{\lambda_1}{\mu_1} = \frac{P_1\lambda}{P_0\mu_1}, \rho_2 = \frac{\lambda_2}{\mu_2} = \frac{P_2\lambda}{P_0\mu_2} \qquad \text{(5points)}$$

By Jackson's Theorem,
the occupancy distribution is

$$P(n_0,n_1,n_2) = \rho_0^{n_0}(1-\rho_0)\rho_1^{n_1}(1-\rho_1)\rho_2^{n_2}(1-\rho_2) \qquad \text{(5points)}$$

(b) The equivalent tandem system is as follows:



(5points)

$$T = \frac{\rho_0}{\lambda(1-\rho_0)} + \frac{\rho_1}{\lambda(1-\rho_1)} + \frac{\rho_2}{\lambda(1-\rho_2)}$$

$$= \frac{\frac{\lambda}{P_0\mu_0}}{\lambda(1-\frac{\lambda}{P_0\mu_0})} + \frac{\frac{P_1\lambda}{P_0\mu_1}}{\lambda(1-\frac{P_1\lambda}{P_0\mu_1})} + \frac{\frac{P_2\lambda}{P_0\mu_2}}{\lambda(1-\frac{P_2\lambda}{P_0\mu_2})}$$

$$= \frac{S_0}{1-\lambda S_0} + \frac{S_1}{1-\lambda S_1} + \frac{S_2}{1-\lambda S_2},$$

where $S_0 = \frac{1}{P_0\mu_0}, S_1 = \frac{P_1}{P_0\mu_1}, S_2 = \frac{P_2}{P_0\mu_2}$ (7points)

3. (10 points) Consider the following argument in the M/G/1 system: When a customer arrives, the probability that another customer is being served is $\lambda\bar{X}$. Since the served customer has mean service time $\bar{X}$, the average time to complete the service is $\bar{X}/2$. Therefore, the mean residual service time is $\lambda\bar{X}^2/2\ (=\lambda\bar{X}\times\bar{X}/2)$. What is wrong with this argument?

Sol:

The problem with the argument given is that more customers arrive while long-service customers are served, so the average service time of a customer found in service by another customer upon arrival is more than E{X}. (10points)

4. (20 points) Indicate the correct answer to each of the following topics, and briefly explain why.

(a) (5 points) A customer arriving to an M/G/1 system in steady-state sees a system that is statistically identical to the one seen by an observer looking at the same system at an arbitrary time.

A. True

B. False

(b) (5 points) An irreducible, aperiodic, discrete-time Markov chain that is in steady-state is time-reversible.

A. True

B. False

(c) (5 points) Consider two tandem transmission lines of equal capacity where Poisson arrivals of rate λ packets/sec enter the first queue and where packet lengths are exponentially distributed and are independent of each other as well as the inter-arrival times at the first queue, then,

A. Both queues can be modeled as M/M/1.

B. The first queue is M/M/1, but the second queue can not be modeled as M/M/1.

(d) (5 points) The maximum achievable throughputs of slotted ALOHA, unslotted ALOHA, splitting algorithm, CSMA slotted ALOHA, and CSMA/CD can be ordered as follows: unslotted ALOHA < slotted ALOHA < splitting algorithm < CSMA slotted ALOHA < CSMA/CD. Assume that the propagation delay is much shorter than the average packet transmission time.

A. True

B. False

(a) A. True (5points)

In the steady-state, if arrival process is Poisson,
then the system is statistically identical as seen by an arriving
or a departing customer or by an observing looking at the system at random time.

(b) B. False (5points)

A Markov chain is said to be time-reversible if $\pi_i P_{ij} = \pi_j P_{ji}$ for all ij.
Irreducible and aperiodic are not enough to guarantee the time-reversible.

(C) B. The first queue is M/M/1, but the second queue can not be modeled as M/M/1. (5points)

The interarrival times at the second queue are strongly correlated with the packet lengths.
In particular, the interarrival time of two packetes at the second queue is
greater than or equal to the transmission time of the second packet at first queue.
As a result, long packets will typically wait less time at the second queue than short packets,
since their transmission at the first queue takes longer,
thereby giving the second queue more time to empty out.

(d) A. True (5points)

unslotted ALOHA: 1/(2e),	slotted ALOHA: 1/e,
splitting algorithm: 2/3,	CSMA slotted ALOHA: $1/(1+\sqrt{2\beta})$,
slotted CSMA/CD: $1/(1+3.31\beta)$	unslotted CSMA/CD: $1/(1+6.2\beta)$

5. (25 points) We consider slotted CSMA with variable packet lengths. Assume that the time to transmit a packet is a random variable *X*; for consistency with the slotted assumption in the textbook, assume that *X* is discrete, taking values that are integer multiples of β. Assume that all transmissions are independent and identically distributed (i.i.d.) with mean $\bar{X} = 1$.

(a) Let $g(n) = \lambda\beta + q_r n$ be the expected number of attempted transmissions following a state transition to state *n*, and assume that this number of attempts is Poisson with mean $g(n)$. Ignoring collisions of more than two packets as negligible, show that the expected time between state transitions in state *n* is at most

$\beta e^{-g(n)} + (1+\beta)g(n)e^{-g(n)} + (1+\beta/2)g^2(n)e^{-g(n)}$. (15 points)

(b) Find a lower bound to the throughput (i.e., the expected number of departures per unit time) in state *n* as well as its approximate maximum value for small β. (10 points)

(a)

The time between state transitions is

β with probability $e^{-g(n)}$,

$(1+\beta)$ with probability $g(n)e^{-g(n)}$,

and at most $(2+\beta)$ with probability $\dfrac{g^2(n)}{2}e^{-g(n)}$ (10 points)

(ignoring collisions of more than two packets).

Thus the expected time between transitions is at most

$\beta e^{-g(n)} + (1+\beta)g(n)e^{-g(n)} + (1+\beta/2)g^2(n)e^{-g(n)}$ (5 points)

(the expected time between transitions is

$$= \sum_{k=0}^{\infty} (\beta + k)\frac{g^k(n)}{k!}e^{-g(n)}$$

$$\simeq \sum_{k=0}^{2} (\beta + k)\frac{g^k(n)}{k!}e^{-g(n)}$$ (ignoring collisions of more than two))

(b)

The success probability in state n is $g(n)e^{-g(n)}$,
so the expected number of departures per unit time is
the ratio of this to expected time between transitions
(this can be justified rigorously by renewal theory).

Thus the expected number of departures per unit time is at least

$$\frac{g(n)e^{-g(n)}}{\beta e^{-g(n)} + (1+\beta)g(n)e^{-g(n)} + (1+\beta/2)g^2(n)e^{-g(n)}} = \frac{g(n)}{\beta + (1+\beta)g(n) + (1+\beta/2)g^2(n)}$$ (5points)

Taking the derivative of this with respect to g(n),
we find a maximum where $g^2(n) = \beta/(1+\beta/2)$.
Thus for small β, g(n) is approximately the square root of β.

Substituting this back into the expression $\frac{g(n)}{\beta + (1+\beta)g(n) + (1+\beta/2)g^2(n)}$,
the maximum throughput (i.e., departures per unit time), is approximately

$$\frac{\sqrt{\beta}}{\beta + (1+\beta)\sqrt{\beta} + \beta} \simeq \frac{1}{(1+\sqrt{\beta})^2} \simeq 1\text{-}2\sqrt{\beta}$$ (5points)