SIGNAL and SYSTEMS

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EXAM I

[Problem 1] (20 points) (1) Prove that time-invariant linear discrete-time systems satisfy the following property :

$$(x[n]*h_1[n])*h_2[n] = (x[n]*h_2[n])*h_1[n] \qquad (1.1)$$

where x[n] is the input, while $h_k[n]$ is the impulse response of the time-invariant linear discrete-time system H_k , k=1,2. (2) Also, give an illustrative example showing that the property in (1.1) does not hold if even one of the two systems H_k , k=1,2 is not linear.

[Problem 2] (30 points) (1) Strictly speaking, the following general Nth-order linear constant-coefficient ODE cannot be used to represent a causal system S. Explain why.

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k} \qquad (2.1)$$

(2) Show that the system represented by (2.1) is linear and time-invariant.

(3) Find the general Nth-order linear constant-coefficient ODE which can represent legally a causal system but which gives the same input-output transfer function as the above ODE.

(4) Give the block diagram representation of the Nth-order linear constant-coefficient ODE you have found in (3).

[Problem 3] (10 points) Consider an LTI system S relaxed at $t = -\infty$ and an input signal $x(t) = 4e^{-2t}u(t-3)$. If

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) \qquad (3.1)$$

and

$$\frac{dx(t)}{dt} \rightarrow \boxed{h(t)} \rightarrow -2y(t) + 3e^{-3t}u(t) \qquad (3.2)$$

then determine the impulse response h(t) of S.

[Problem 4] (10 Points) Suppose we are given the following information about a signal x[n] where N is the period of signal and the Fourier series coefficient of the signal is denoted by a_k .

1. x[n] is a real and odd signal.
2. N = 7
3. $-2a_8 = a_{12}$
4. $|a_{15}|^2 = 1$
5. $\frac{1}{7}\sum_{n=0}^{6}|x[n]|^2 = 10$

Specify two different signals that satisfy these conditions

[Problem 5] (30 Points) Consider a discrete-time LTI system S with the following impulse response.

$$h[n] = \begin{cases} 1 & 0 \le n \le 2 \\ -1 & -2 \le n \le -1 \\ 0 & \text{otherwise} \end{cases} \tag{5.1}$$

Suppose that the input to the system S is the periodic signal given gy

$$x[n] = \sum_{k=-\infty}^{\infty} 2\delta(n-4k) \tag{5.2}$$

With period $T = 4$.

(1) Determine the Fourier-series coefficients of the output $y[n]$,

(2) The signal $x_{(m)}[n]$ defined as

$$x_{(m)}[n] = \begin{cases} x[\frac{n}{m}] & n = 0, \pm m, \pm 2m, \ldots \\ 0 & \text{otherwise} \end{cases} \tag{5.3}$$

is obtained by scaling the signal $x[n]$ in time. Let a_k be the Fourier-series coefficient of the input signal $x[n]$, and b_k be the Fourier-series coefficient of the time-scaling signal $x_{(m)}[n]$. Derive the relation between a_k and b_k.

(3) Suppose that the input $x[n]$ to the discrete-time LTI system S is changed into $x_{(5)}[n]$. Determine Fourier-series coefficients of the output $y[n]$

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