SIGNAL and SYSTEMS

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EXAM III

[Problem 1] (20 points) A discrete-time system is implemented as shown in **Fig. 1**. The system S shown below is an LTI system with impulse response $h_{lp}[n]$ which is relaxed at $n = -\infty$.

(a) Show that the overall system is time invariant.
(b) If $h_{lp}[n]$ is a low-pass filter, what type of filter does the system in **Fig. 1** implement?

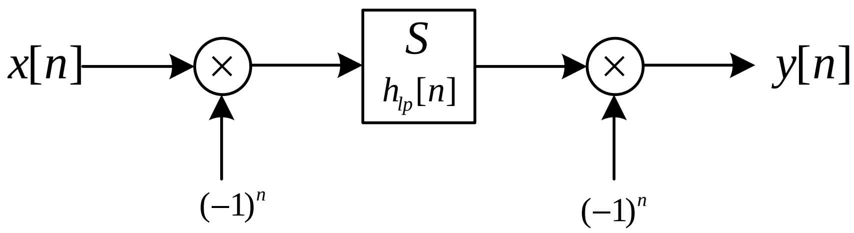

Fig. 1

[Problem 2] (20 points) In this problem, we explore some of the filtering issues involved in the commercial version of a typical system that is used in most modern cassette tape decks to reduce noise. The primary source of noise is the high-frequency hiss in the tape playback process, which, in some part, is due to the friction between the tape and the playback head. Let us assume that the noise hiss that is added to the signal upon playback has the spectrum of **Fig. 2** when measured in decibels, with 0dB equal to the signal level at 100 Hz. The spectrum $S(j\omega)$ of the signal has the shape shown in **Fig. 3**.

The system that we analyze has a filter $H_1(j\omega)$ which conditions the signal $s(t)$ before it is recorded. Upon playback, the hiss $n(t)$ is added to the signal. The system is represented schematically in **Fig. 4**.
Suppose we would like our overall system to have a signal-to-noise ratio of 40dB over the frequency range 50 Hz< $\omega / 2\pi$ <20 kHz.
(a) Determine the transfer characteristic of the filter $H_1(j\omega)$. Sketch the Bode plot of $H_1(j\omega)$.
(b) If we were to listen to the signal $p(t)$, assuming that the playback process does nothing more than add hiss to the signal, how do you think it would sound?

(c) What should the Bode plot and transfer characteristic of the filter $H_2(j\omega)$ be in order for the signal $\hat{s}(t)$ to sound similar to $s(t)$?

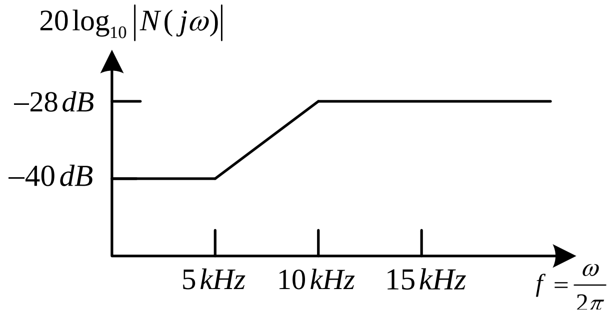

Fig. 2

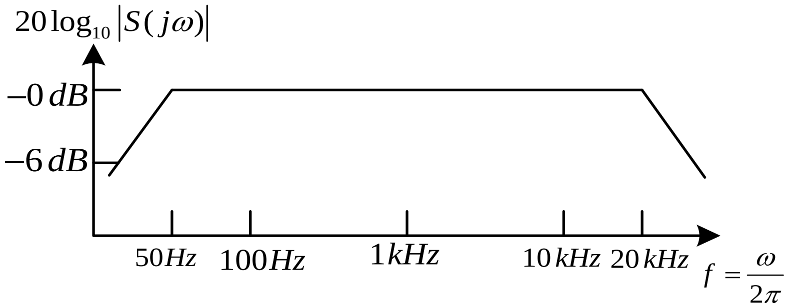

Fig. 3

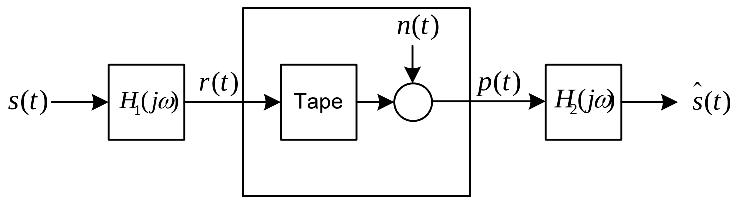

Fig. 4

[Problem 3] (10 points) Show that a discrete-time signal $x[n]$ with bandwidth $\omega_M = \frac{\pi}{3}$ can be recovered from the sampled sequence $x[3k],\ k=0,\ \pm1,\ \pm2,\ \cdots$ such that

$$x[n] = \sum_{k=-\infty}^{\infty} x[3k]\left(\frac{\sin\left(\frac{\pi}{3}(n-3k)\right)}{\frac{\pi}{3}(n-3k)}\right) \tag{3.1}$$

[Problem 4] (10 points) Let $x[n]$ be a causal sequence (i.e. if $x[n]=0, n<0$).

(a) Derive $x[0]$ and $x[1]$ from its z transform $X(z)$.
(b) If $x[0]$ is a nonzero and finite signal, show that there are no poles or zeros of $X(z)$ at z=∞.
(c) Using the result of (a), determine $x[1]$ from

$$X(z)=\frac{1-\frac{1}{3}z^{-1}}{(1-z^{-1})(1+2z^{-1})}, |z|>2 \tag{4.1}$$

(d) Determine $x[1]$ from (4.1) using the inverse z-transform and compare the result with that of (c).

[Problem 5] (20 points) The system in **Fig. 5** processes continuous-time signals using a discrete- time filter $h_d[n]$. Suppose that the discrete-time filter $h_d[n]$ satisfies the following equation

$$y_d[n]=-\frac{1}{2}y_d[n-1]+x_d[n] \tag{5.1}$$

Suppose that the input signal $x_c(t)$ is band limited such that $X_c(j\omega)=0$ for $|\omega|>\frac{\pi}{T}$.

(a) Determine the frequency response $H_c(j\omega)$ of equivalent overall system with input $x_c(t)$ and output $y_c(t)$.
(b) Determine the inverse Fourier transform of $H_c(j\omega)$.

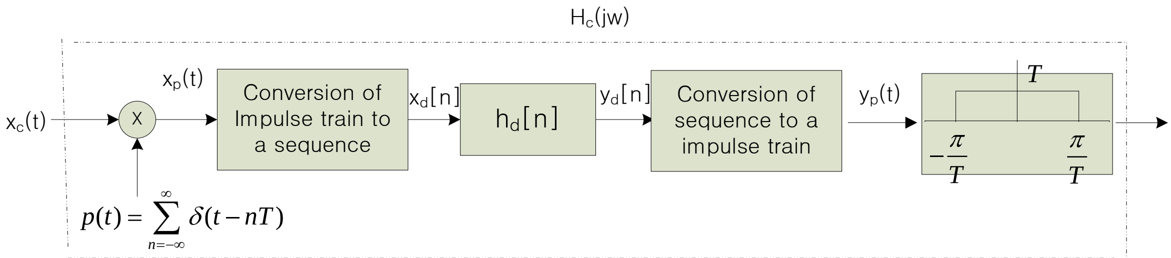

Fig. 5

[Problem 6] (20 points) Consider the following general Nth-order linear constant-coefficient difference equation.

$$\sum_{k=0}^{N}a_k y[n-k]=\sum_{k=0}^{M}b_k x[n-k] \tag{6.1}$$

(a) Show that if N and M are positive integers, then the difference equation in (6.1) always represents a causal system S.
(b) Give the block-diagram representation of the difference equation in (6.1), in which only the delay elements of $x[n]$ are not involved

(c) Using the result of (b), find the vector-form representation of the difference equation in (6.1) as follows.

$$
\begin{aligned}
z[n] &= \Phi z[n-1] + \Gamma x[n] \\
y[n] &= H z[n]
\end{aligned}
\qquad (6.2)
$$

where Φ, Γ, and H are matrices with appropriate dimensions.

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