# Topics in Communications (Spring, 2008), Final Exam

1. Consider the following problem

$$\text{minimize} \quad x^T x$$
$$s.t. \;\; Ax \le b$$

with $x \in R^{1000}$, $A \in R^{10 \times 1000}$ and answer the questions. We assume $A$ is full rank. (a) What is the minimizer? (b) Express the primal problem as a dual problem. (c) Does strong duality hold? (d) Suppose $\lambda^*$ is optimal for the dual problem. Can we find a primal optimal point? If so, is it unique? (e) Compare the number of variables for the primal and dual problems?

2. Consider the following problem

$$\text{minimize} \quad \int_{-\infty}^{C^T x} \frac{1}{\sqrt{2\pi}} e^{t^2/2} dt$$
$$s.t. \quad Ax \leqq b, \;\; Hx = g$$

(a) Is it a convex problem? (b) Is it a quaiconvex problem? (c) Can we find an equivalent problem in LP?

3. We consider a wireless system that uses time-domain access(TDMA) to support $n$ communication flows. The flows have (nonnegative) rates $r_1, \cdots, r_n$, given in bits/sec. To support a rate $r_i$ on flow $i$ requires transmitter power

$$p = a_i (e^{br} - 1),$$

where $b$ is a (known) positive constant, and $a_i$ are (known) positive constants related to the noise power and gain of receiver $i$. Time is divided up into periods of some fixed duration $T$ (seconds). Each of these $T$-long periods is divided into $n$ time-slots, with durations $t_1, \cdots, t_n$ that must satisfy $t_1 + t_2 + \cdots + t_n = T$, $t_i \geqq 0$. In time-slot $i$, communications flow $i$ is transmitted at an instantaneous rate $t = Tr_i / t_i$, so that over each $T$-long period, $Tr_i$ bits from flow $i$ are transmitted. The power required during time-slot $i$ is $a_i (e^{bTr_i/t_i} - 1)$, so the average transmitter power over each $T$-long period is

$$P = (1/T) \sum_{i=1}^{n} a_i t_i (e^{bTr_i/t_i} - 1).$$

When $t_i$ is zero, we take $P = \infty$ if $r_i > 0$, and $P = 0$ if $r_i = 0$. (The latter corresponds to the case when there is zero flow, and also, zero time allocated to the flow.)

The problem is to find rates $r \in \mathbb{R}^n$ and time-slot durations $t \in \mathbb{R}^n$ that maximize the log utility function

$$U(r) = \sum_{i=1}^{n} \log r_i,$$

Subject to $P \leqq P^{\max}$. (This utility function is often used to ensure 'fairness'; each communication flow gets at least some positive rate.) the problem data are $a_i$, $b$, $T$ and $P^{\max}$; the variables are $t_i$ and $r_i$.

(a) Formulate this problem as a convex optimization problem. Feel free to introduce new variables, if needed, or to change variables. Be sure to justify convexity of the objective or constraint function in your formulation.

(b) Give the optimality conditions for your formulation. Of course simpler optimality conditions are preferred to complex ones.(You don't need to solve it.)

4. Derive the KKT conditions for the problem

$$\begin{array}{ll} \text{minimize} & \operatorname{tr} X - \log\det X \\ \text{subject to} & Xs = y \end{array}$$

with variable $X \in S^n$ and domain $S^n_{++}$. $y \in \mathbb{R}^n$ and $s \in \mathbb{R}^n$ are given, with $s^T y = 1$. Verify that the optimal solution is given by

$$X^* = I + yy^T - \frac{1}{s^T s} s s^T$$

5. *Gradient and Newton methods for composition functions.* Suppose $\phi : \mathbb{R} \to \mathbb{R}$ is increasing and convex, and $f : \mathbb{R}^n \to \mathbb{R}$ is convex, so $g(x) = \phi(f(x))$ is convex. (We assume that $f$ and $g$ are twice differentiable.) The problems of minimizing $f$ and minimizing $g$ are clearly equivalent.

Compare the gradient method and Newton's method, applied to $f$ and $g$. How are the search directions related? How are the methods related if an exact line search is used?

Important Notice

A. Time: 2:30 ~ 4:30

B. Only textbook is allowed (No other books, documents, or papers)

C. Extra answer sheets will be used.