# Communication Systems

(Final Exam Solution(6/14))

1.

a)

- Orthogonal Frequency Division Multiplexing
- Minimum Shift Keying
- Offset Quadrature Phase Shift Keying
- Staggered Quadrature Phase Shift Keying
- Continuous Phase Frequency Shift Keying

b)

- Can easily adapt to severe channel conditions without complex equalization
- Robust against narrow-band co-channel interference
- Robust against intersymbol interference and fading caused by multipath propagation
- High spectral efficiency
- Efficient implementation using FFT
- Low sensitivity to time synchronization errors
- Tuned sub-channel receiver filters are not required
- Facilitates single frequency networks, i.e. transmitter macrodiversity

2.

a) Correction of burst error

b) If we take all sequences of length $n$ that are orthogonal to all vectors of this $k$-dimensional linear subspace, the result will be an $(n-k)$-dimensional linear subspace called the orthogonal complement of the $k$-dimensional subspace. This $(n-k)$-dimensional linear space naturally defines an $(n, n-k)$ linear code which is known as the dual of the original $(n,k)$ code of $C$.

3. The capacity of the channel is defines as

$$C = \max_{p(x)} I(X;Y) = \max_{p(x)} \left[ H(Y) - H(Y|X) \right]$$

The conditional entropy $H(Y|X)$ is

$$H(Y|X) = p(X=a)H(Y|X=a) + p(X=b)H(Y|X=b) + p(X=c)H(Y|X=c)$$

However,

$$\begin{aligned} H(Y|X=a) &= -\sum_k p(Y=k|X=a)\log P(Y=k|X=a) \\ &= -(0.2\log 0.2 + 0.3\log 0.3 + 0.5\log 0.5) \\ &= H(Y|X=b) = H(Y|X=c) = 1.4855 \end{aligned}$$

and therefore

$$H(Y|X) = \sum_k p(X=k)H(Y|X=k) = 1.4866$$

Thus,

$$I(X;Y) = H(Y) - 1.4855$$

To maximize $I(X;Y)$, it remains to maximize $H(Y)$. However, $H(Y)$ is maximized when $Y$ is a uniformly distributed random variable, if such a distribution can be achieved by an appropriate input distribution. Using the symmetry of the channel, we observe that a uniform input distribution produces a uniform output.

Thus, the maximum of $I(X;Y)$ is achieved when $p(X=a)=p(X=b)=p(X=c)=\frac{1}{3}$ and the channel capacity is

$$C=\log_2 3-H(Y|X)=0.995 \text{ bits/transmission}$$

4. If $\mathbf{c}$ is a codeword, then $w(\mathbf{c})=d(\mathbf{c}_i,0)$.

$$\begin{aligned} d_{\min} &= \min_{\substack{\mathbf{c}_i,\mathbf{c}_j \\ i\neq j}} d(\mathbf{c}_i,\mathbf{c}_j) \\ &= \min_{\substack{\mathbf{c}_i,\mathbf{c}_j \\ i\neq j}} w(\mathbf{c}_i-\mathbf{c}_j) \\ &= \min_{\substack{\mathbf{c}_l \\ \mathbf{c}_l\neq 0}} w(\mathbf{c}_l) \\ &= w_{\min} \end{aligned}$$

5.

a) Linear
b) (5,2)
c) <Generator matrix>

We have to find the codewords corresponding to information sequences 10 and 01 which are orthogonal.

These are 10100 and 01111, respectively.

Therefore, we have

$$\mathbf{G}=\begin{bmatrix}10100\\01111\end{bmatrix}$$

It is seen that for the information sequence $(x_1, x_2)$, the codeword is given by

$$(c_1, c_2, c_3, c_4, c_5) = (x_1, x_2)\mathbf{G}$$

or

$$c_1 = x_1$$

$$c_2 = x_1$$

$$c_3 = x_1 \oplus x_2$$

$$c_4 = x_2$$

$$c_5 = x_2$$

<Parity check matrix>

Here

$$\mathbf{G} = \begin{bmatrix} 10100 \\ 01111 \end{bmatrix}$$

$$\mathbf{I} = \begin{bmatrix} 10 \\ 01 \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} 100 \\ 111 \end{bmatrix}$$

Nothing that in the binary case $-\mathbf{P}^t = \mathbf{P}^t$, we conclude that

$$\mathbf{P}^t = \begin{bmatrix} 11 \\ 01 \\ 01 \end{bmatrix}$$

and, therefore,

$$\mathbf{H} = \left[\begin{array}{c|c} 11 & 100 \\ 01 & 010 \\ 01 & 001 \end{array}\right]$$

d) <standard array>

The generator polynomial of the code is given by

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

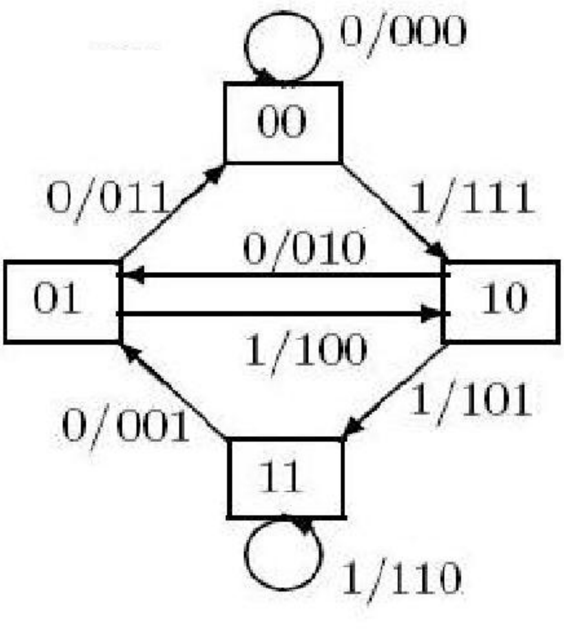
And the parity check matrix corresponding to **G** is given by

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

We obtain the standard array

| | | | |
|---|---|---|---|
| 00000 | 01111 | 10100 | 11011 |
| 10000 | 11111 | 00100 | 01011 |
| 01000 | 00111 | 11100 | 10011 |
| 00010 | 01101 | 10110 | 11001 |
| 00001 | 01110 | 10101 | 11010 |
| 11000 | 10111 | 01100 | 00011 |
| 10010 | 11101 | 00110 | 01001 |
| 10001 | 11110 | 00101 | 01010 |

6.



7.

MSK - a special case of CPFSK (h=1/2), better bandwidth efficiency, no spectral lobe, continuous waveform

OQPSK - easier modulation, discontinuous waveform, phase change $\pm 90^\circ$

QPSK - phase change $\pm 90^\circ, \pm 180^\circ$

MSK is a kind of frequency modulation. QPSK is a kind of phase modulation.

8.

a) A maximum-length shift-register sequence has a length $L = 2^m - 1$ bits and is generated by an $m$-stage shift register with linear feedback.

b) If the frequency-hopping rate, $R_h$, is equal to or lower than the symbol rate, the FH system is called a slow frequency-hopping spread-spectrum system.

c) To reduce interference from transmissions in adjacent cells, different sets of frequencies are assigned to adjacent base stations and frequencies are reused according to some design plan. The frequency reuse factor is the rate at which the same frequency can be used in the network.