Quiz #2 solution

1. A positive point charge Q is located at distances d_1 and d_2, respectively, from two grounded perpendicular conducting half-planes, as shown in Fig 1. Determine the force on Q caused by the charges induced on the planes.

Sol)

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

where,

$$\mathbf{F}_1 = -\mathbf{a}_y \frac{Q^2}{4\pi\varepsilon_0 (2d_2)^2},$$

$$\mathbf{F}_2 = -\mathbf{a}_x \frac{Q^2}{4\pi\varepsilon_0 (2d_1)^2},$$

$$\mathbf{F}_3 = \frac{Q^2}{4\pi\varepsilon_0 [(2d_1)^2 + (2d_2)^2]^{3/2}} (\mathbf{a}_x 2d_1 + \mathbf{a}_y 2d_2).$$

Therefore,

$$\mathbf{F} = \frac{Q^2}{16\pi\varepsilon_0} \{\mathbf{a}_x [\frac{d_1}{(d_1^2 + d_2^2)^{3/2}} - \frac{1}{d_1^2}] + \mathbf{a}_y [\frac{d_2}{(d_1^2 + d_2^2)^{3/2}} - \frac{1}{d_2^2}]\}. \quad \text{(N)}$$

2. A conducting material of uniform thickness h and conductivity σ has the shape of a quarter of a flat circular washer, with inner radius a and outer radius b, as shown in Fig. 5-8 . Determine the resistance between the end faces.

Sol)

B.C.

$V = 0 \quad at \quad \phi = 0$

$V = V_0 \quad at \quad \phi = \pi / 2$

Laplace eq.

$$\frac{d^2 V}{d\phi^2} = 0$$

General solution

$$V = c_1\phi + c_2$$

Using boundary condition,

$$V = \frac{2V_0}{\pi}\phi \quad \textbf{[V]}$$

Current density is,

$$J = \sigma E = -\sigma\nabla V = -\hat{\phi}\sigma\frac{\partial V}{r\partial\phi} = -\hat{\phi}\frac{2\sigma V_0}{\pi r} \quad \textbf{[A/m}^2\textbf{]}$$

The total current is

$$I = \int_S J\cdot ds = \frac{2\sigma V_0}{\pi}h\int_a^b\frac{dr}{r} = \frac{2\sigma h V_0}{\pi}\ln\frac{b}{a} \quad \textbf{[A]}$$

Therefore,

$$R = \frac{V_0}{I} = \frac{\pi}{2\sigma h\ln(b/a)} \quad [\Omega]$$