Quiz #3 Solution

Prove that,

$$\int_V \left(\nabla \times \vec{F}\right) dv = -\oint_S \vec{F} \times ds$$

(sol)

Apply the divergence theorem to $\left(\vec{F} \times \vec{C}\right)$, where $\vec{C}$ is a constant vector.

$$\int_V \nabla \bullet \left(\vec{F} \times \vec{C}\right) dv = \oint_S \left(\vec{F} \times \vec{C}\right) \bullet d\vec{s} \qquad \textbf{(3 point)}$$

From problem P.2-33,

$$\nabla \bullet \left(\vec{F} \times \vec{C}\right) = \vec{C} \bullet \left(\nabla \times \vec{F}\right) - \vec{F} \bullet \left(\nabla \times \vec{C}\right) = \vec{C} \bullet \left(\nabla \times \vec{F}\right)$$
$$\left(\because \vec{C} \text{ is constant } \rightarrow \nabla \times \vec{C} = 0\right) \qquad \textbf{(3 point)}$$

and from Eq. (2-19),

$$\left(\vec{F} \times \vec{C}\right) \bullet d\vec{s} = -\vec{C} \bullet \left(\vec{F} \times d\vec{s}\right) \quad \textbf{(2 point)}$$

$$\therefore \vec{C} \bullet \int_V \left(\nabla \times \vec{F}\right) dv = -\vec{C} \bullet \oint_S \left(\vec{F} \times d\vec{s}\right) \Rightarrow \int_V \left(\nabla \times \vec{F}\right) dv = -\oint_S \left(\vec{F} \times d\vec{s}\right) \quad \textbf{(2 point)}$$