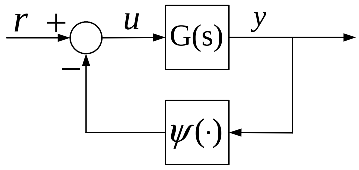
Nonlinear Systems Final-Exam. (Open-book, 2005 / 2)

1. Is it possible for a 2-dimensional autonomous ODE to have exactly two equilibria, both of which are asymptotically stable, such that all trajectories converge to one of the other equilibrium?

2. Based on the (graphical) circle criterion, state a condition for absolute stability of the following closed-loop system when the static nonlinearity ψ lies in the sector $[k_1, k_2]$ where $k_1 < k_2 < 0$. In the figure, assume that $r = 0$ and $G(s)$ is Hurwitz.



3. Assuming that $|a| \geq \mu > 0$, solve the following.

   (a) Show that
   $$\left\| \frac{1}{s^2 + 2\mu s + \mu^2 + a^2} \right\|_\infty = \frac{1}{2\mu|a|}.$$

   (b) Consider the system
   $$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -(\mu^2 + a^2)x_1 - 2\mu x_2 + q\cos(\omega t)|x_1|. \end{aligned}$$
   Using the small-gain theorem, find a condition about q, ω, μ, and a that guarantees the UGAS of the origin.

4. Consider the system
$$\begin{aligned} \dot{z} &= Az + Bx, \qquad z \in \mathbb{R}^n, \\ \dot{x} &= Cz + Dx + u, \qquad x \in \mathbb{R},\ u \in \mathbb{R}, \\ y &= x, \qquad y \in \mathbb{R}, \end{aligned}$$
where A is Hurwitz. Assuming that the above system is not passive, show that the feedback control
$$u = -\gamma y + v$$
can make the closed-loop system strictly passive from v to y if the constant γ is sufficiently large.

5. Make your own question and answer it. Or, you may write your own finding, your correction to the textbook or handouts, or your own theory. (Recommendation: Uniqueness, rigorousness, and importance of the issue will help to get higher points.)