Nonlinear Systems Final Exam. (Open book, 2006 / 2)

1. (10pts) Using the averaging theory, describe the behavior of the system

$$\begin{aligned}\dot{x}_1 &= x_2,\\ \dot{x}_2 &= -x_2 - \sin(\omega t)(x_1 + \sin(\omega t))^2,\end{aligned}$$

   where ω is relatively large positive constant.

2. (10pts) Using the passivity theorem, prove that the origin of the system

$$\begin{aligned}\dot{x}_1 &= -f(x_1) + x_2\\ \dot{x}_2 &= -g(x_2) - x_1^3\end{aligned}$$

   where the functions f and g are in the sector $(0, \infty)$, is globally asymptotically stable.

3. (15pts) Using the small gain theorem, prove that the origin of the system

$$\begin{aligned}\dot{x}_1 &= -x_1^3 + x_2\\ \dot{x}_2 &= -x_2 + \frac{1}{2}x_1^3\end{aligned}$$

   is globally asymptotically stable.

4. (15pts) Using the center manifold theory, provide a sufficient condition on the parameter, such that the origin of the following system is locally asymptotically stable:

$$\begin{aligned}\dot{w} &= wv + aw^3 + bwv^2\\ \dot{v} &= -v + cw^2.\end{aligned}$$

5. (10pts) Using the singular perturbation, determine the stability of the origin of the system

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -x_3\\ \dot{x}_3 &= -kx_3 + k\sin x_1 + kx_2\end{aligned}$$

   if k is relatively large enough.

6. (10pts) Find $\alpha > 0$, as large as you can, such that the following system is absolutely stable with the sector $(-\alpha, \infty)$:

$$G(s) = \frac{1}{s^2 + 2s + 1}.$$