Eng Math2. Mid Term (10/29/2008)

(Closed book and note: 90 min. Total 200)

1. Evaluate the following integral (you need to give proper reasoning):

$$\int_0^\infty \frac{\sin w}{w}\,dw$$

2. It has been well known that surface integration of electric field over a closed surface results in q/ε_0 (Gaussian law) where q is point charge enclosed by the surface, and ε_0 is the permittivity of free space: $\oint_S \vec{E}\bullet\vec{n}da = \frac{q}{\varepsilon_0}$. It is also well known that $\vec{F}(\text{force}) = q\vec{E}$ and $\varphi(\text{electrostatic potential}) = -\int_{\vec{r}_{ref}}^{\vec{r}} \vec{E}\bullet d\vec{r}$.

Evaluate the followings:

(a) $\oint \nabla\varphi\bullet d\vec{r} = ?$

(b) $\nabla\bullet\vec{E} = ?$

(c) $\nabla\times\vec{E} = ?$

3. Answer to each question properly:

(a) simplify $\frac{d}{dt}\left[\vec{r}(t)\bullet\left(\vec{r}\,'(t)\times\vec{r}\,''(t)\right)\right]$.

(b) Evaluate the integral $\oint_C (zdx + xdy + ydz)$ where C is the trace of the cylinder $x^2+y^2=1$ in the plane $y+z=2$. Orient C counterclockwise as viewed from above.

4. Answer to each question properly:

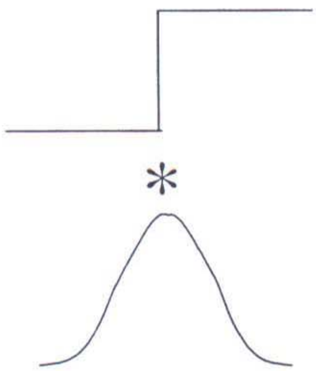
(a) Find the Fourier transform of $\frac{d}{dx}\left[f(x)*g(x)\right]$.

(b) Find the inverse Fourier transform of 1.

5. Find the Fourier cosine transform of $f(x)$.

$$f(x)=e^{-ax}$$

6. (a) Referring to the figure on the right, <u>sketch a rough shape</u> of the convolution of a step function and a Gaussian function.



Hint)

$$u(x)*g(x)=\int_{-\infty}^{\infty}u(p)g(x-p)dp,$$
$$u(x): 1 \ when \ x \geq 0, \ 0 \ when \ x<0.$$
$$g(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$$

(b) Given that $\hat{f}[e^{-x^2}]=\frac{1}{\sqrt{2}}e^{-\frac{w^2}{4}}$, evaluate the Fourier transform of $(x-a)e^{-(x-a)^2}$.

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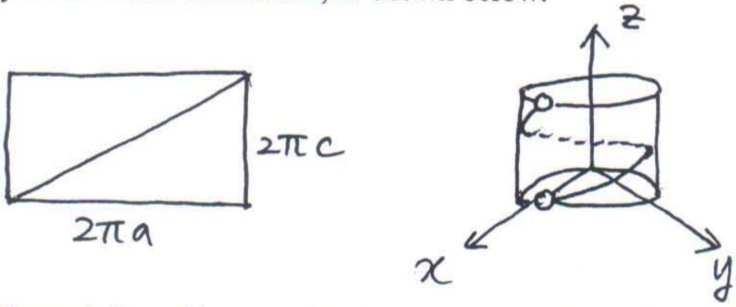
Draw a diagonal straight line on a paper with rectangular shape and roll up the paper around a cylinder with a radius of a, as shown below:



The curve formed from this operation becomes a circular helix, which can be expressed as follows:

$$r(t) = (a \cos t, a \sin t, ct)$$

Obtain the direction, tangential vector, and the length of the curve (circular helix).

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If $r = \sqrt{x^2 + y^2 + z^2}$ in the axisymmetric spherical coordinate,

(a) Obtain $\nabla \varphi(r)$

(b) Prove that $\nabla^2 \varphi(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \varphi}{\partial r}\right)$

(c) Obtain $\nabla^2 (1/r)$

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A vector field F is defined at all the points in xy-plane except the origin as shown below:

$$\underline{F} = \left(-\frac{y}{x^2 + y^2}\right)\hat{\imath} + \left(\frac{x}{x^2 + y^2}\right)\hat{\jmath}$$

(a) Obtain $\nabla \times \underline{F}$

(b) Obtain the line integral $\oint_c \underline{F} \cdot dr$

for a circle with a radius of 1 around the origin.

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Express the Fourier transform of the following functions

(a) $\mathcal{F}[f(x-a)]$

(b) $\mathcal{F}\left[\int_0^x f(v)dv\right]$

in terms of Fourier transform of f(x) $\mathcal{F}[f(x)]$