1. (10pts) Let us assume w is the velocity of propagation for a de Broglie wave of a particle of momentum p and total energy E. Then we obtain $w = \lambda v = (h/p)(E/h) = E/p$ Assuming the particle is moving at nonrelativistic velocity, we find $w = E/p = (mv^2/2)/mv = v/2$. This result means the propagation velocity of the de Broglie wave of the particle is only half of the particle velocity and thus seems disturbing because the matter wave should keep up with the particle. What is wrong for this derivation?

2.(10pts) Consider an electron trapped in a one-dimensional box with infinitely high potential. The box extends from $x = 0$ to $x = a$. The electron has the mass m and the total energy E.

(a) (5pts) Find the energy eigenvalue of the electron by solving Schrödinger equation,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0$$

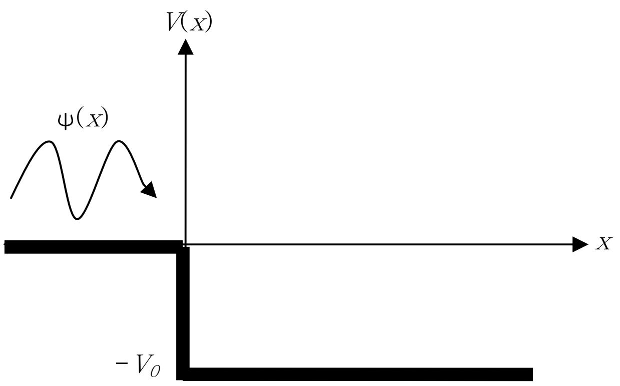
(b) (5pts) If the lowest energy possible for the electron is 10.00 eV, what are the two higher energies the particle can have?

3. (20pts) An electron of mass m and kinetic energy $\boldsymbol{E > 0}$ approaches a potential drop V_0

(a) (5pts) Write down the equations of two regions ($x \geq 0$, $x \leq 0$) from time-independent Schrödinger equation.

(b) (10pts) Assume there is no incoming wave to the right, so you can get three amplitude constants. Let A : incident amplitude, B : reflected amplitude, C : transmitted amplitude.

Determine the reflection coefficient R. ($R = \frac{|B|^2}{|A|^2}$)



(c) (5pts) What is R if $E = V_0 / 3$?

Calculate the transmission coefficient T in this case from R + T = 1.

4. (10pts) Explain the reason for the formation of energy bands and energy gaps in crystals by considering free electrons in a periodic potential well (*i.e.*, Kronig-Penny model)

5. (20pts) Answer the following questions.

(a) (10pts) Calculate how much the kinetic energy of a free electron at the corner of the first Brillouin zone of a simple cubic lattice (three dimensions!) is larger than that of an electron at the midpoint of the face.

(b) (10pts) Calculate the main lattice vectors in reciprocal space of an fcc crystal.

6. (10pts) Using the Drude postulation, derive the conductivity, σ as a function of N_f (number of free electron per unit volume), electron charge e, electron mass m, and relaxation time τ. On the basis of this relation, describe the temperature dependence of resistivity in a pure metal.

7. (20pts) Answer the following questions

(a) (10pts) The probability that a certain energy level is occupied by electrons is given by Fermi distribution function. Plot the $F(E)$ versus E for both $T = 0$ K and $T > 0$ K, and then explain their physical meanings. At what temperature can we expect a 10% probability that electrons in silver have an energy which is 1% above the Fermi energy? ($E_F = 5.5$ eV)

(c) ((10pts) Derive the effective mass, m^* given by

$$m^* = \hbar^2 \left(\frac{d^2E}{dk^2}\right)^{-1}$$