SNU MAE Multivariable Control

Exam Fall 2008

Name:

1	/10
2	/15
3	/5
EXTRA	/α
Total	/30 +α

1. [10 pts] Describe why the followings are true.

(a) For $x \in \mathbb{C}^n, x \neq 0$ and $M \in \mathbb{C}^{m \times n}$,

$$\sqrt{\lambda_{min}(M^*M)} \leq \frac{||Mx||_2}{||x||_2} \leq \sqrt{\lambda_{min}(M^*M)}$$

where $||x||_2 = \sqrt{x^T x}$.

(b) For a stable transfer matrix T such that $y(s) = T(s)u(s)$,

$$\sup_{u \neq 0} \frac{||y(t)||_2}{||u(t)||_2} = ||T||_\infty.$$

2. [15 pts] Consider the MIMO system with the 2×2 loop gain transfer matrix.

$$L(s) \;=\; \begin{bmatrix} 1/2 & 0 \\ ks/(s+2) & 0 \end{bmatrix}$$

The loop gain depends on a parameter $k \in \mathbb{R}$.

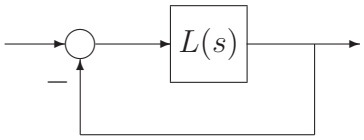
a) Find the ∞-norm of L (the norm still depends on k)

b) For which values of k does the small-gain theorem guarantee that the closed loop of Fig is internally stable?

c) Determine all k for which the closed loop is internally stable.

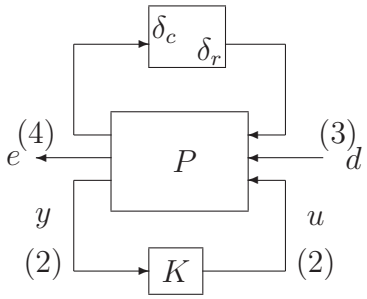
d) Compute $\mu(L(j\omega))$ with respect to structure

$$\Delta \;=\; \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{bmatrix}, \quad \Delta_{ij} \in \mathbb{C}.$$



3. [5 pts] We can use D-K iteration for mu-synthesis. Then, describe why we still need H-infinity control.

4. [EXTRA] Our plant has an uncertainty structure consisting of two scalar elements, one complex (δ_c) and the other real (δ_r). Our performance objective is a H_∞ norm from d to e. The signal d is 3-dimensional, and e is 4-dimensional. The controller takes 2-dimensional measurements and generates 2-dimensional control signals. And suppose we are given a stable controller K that achieves robust stability and robust performance.



Define a ball centered at K as

$$B(K) = \{\tilde{K} : ||\tilde{K} - K||_\infty < 1, \tilde{K} : \text{ stable } 2 \times 2 \text{ transfer function}\}$$

and derive the necessary and sufficient condition such that $K \in B(K)$ is a robustly performing controller. (It is a μ-statement, and you should explain the corresponding block structure and transfer function to be tested.)