# Automata Theory

## Final Exam : 4 December 2008

1. Find a pushdown automaton that accepts

$$L = \{w_1 c w_2 : w_1, w_2 \in \{a, b\}^* \text{ and } w_1 \neq w_2^R\}.$$

2. Find a pushdown automaton that is equivalent to the following grammar.

$$
\begin{aligned}
S &\rightarrow aB \mid bA \mid \epsilon \\
A &\rightarrow a \mid aS \mid bAA \\
B &\rightarrow b \mid bS \mid aBB
\end{aligned}
$$

3. Describe a Turing machine that computes $n!$ when $n$ is given in a unary notation.

4. Explain how the standard Turing machine simulates the 2-tape Turing machine. (Note that both Turing machines should halt on the same set of strings.)

5. Let $L_d = \{\langle M \rangle : \langle M \rangle \in L(M)\}$. Write the definition of $\bar{L}_d$, and prove that $\bar{L}_d$ is not recursively enumerable. Assume that $\Sigma = \{0, 1\}$.

6. Prove that the following language is undecidable.

$$\{\langle M \rangle : L(M) \text{ is a context-free language}\}$$