Elementary Numerical Analysis

2008 년 2 학기

Midterm Examination 1

Oct. 7, 2008

1. Give the answer to the following questions. No need for derivation. (20 points)
   a. The Lagrange interpolation polynomial of order *n*.
   b. The coefficient matrix resulting from the 3rd order least square approximation of *N* data points (*N*>*4*).
   c. The usage of the Hermite polynomial.
   d. The advantages of using orthogonal polynomials in the least square fitting.

2. Determine the following. (35 points)
   a. Binary floating point number representation of 0.40625.
   b. The coefficient of the second order term of the Newton polynomial passing through all the following five points: (0,0), (1,2), (2,3), (3,5), (4,10).
   c. The three *x*-coordinates of the points to be used in the 2nd order interpolation polynomial for $f(x) = sin(x)$ to minimize the maximum interpolation error within $[0, \frac{\pi}{2}]$.
   d. The linear system to be solved to obtain a Pade approximation for $f(x) = e^x$ with $g(x) = \dfrac{a_0 + a_1 x}{1 + b_1 x + b_2 x^2 + b_3 x^3}$.

3. Explain or prove the following as concisely as possible with only essential details. (20 points)

   a. The conditions needed to determine all the coefficients of the cubic spline polynomials given as $p_i(x) = a_i + b_i(x - x_{i-1}) + c_i(x - x_{i-1})^2 + d_i(x - x_{i-1})^3$, $x_{i-1} \le x \le x_i$; $h_i = x_i - x_{i-1}$ for *n+1* data.

   b. The minimax property of the Chebyshev polynomial which states that the normalized Chebyshev polynomial has the minimum max value within the interval [-1,1] among all the normalized polynomials of order *n* passing through (1,1)

4. Consider the zero flux boundary condition applied at the end point in solving one-dimensional particle diffusion equation as shown in the following figure. You want to derive the finite difference approximation (FDA) for the *second* derivative at the last mesh x_n. There can be two ways in deriving the FDA. Answer the following. (15 points)
   a. Derive the usual FDA using the two slopes.
   b. Derive the FDA based on the second order interpolation involving the three points indicated. Give the second order Lagrange interpolation first and then take the second derivative of the polynomial.
   c. The two FDA's derived above are different. Which one would be more accurate? Explain your answer.

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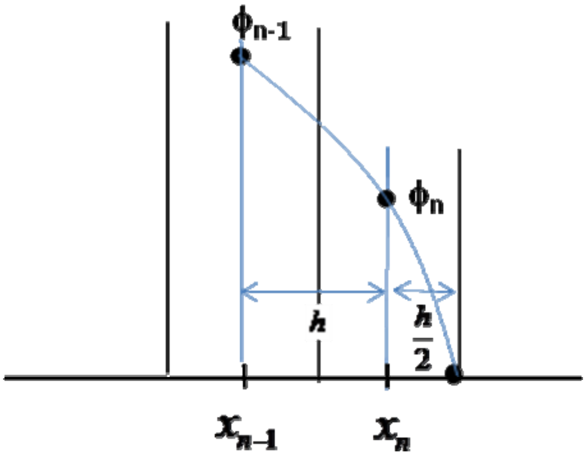

5. Any real function defined in the interval $[a,b]$ can be approximated by a series of orthogonal functions. Namely,

$$f(x)=\sum_{i=0}^{n} a_i\phi_i(x) \quad \text{where} \quad a_i=\frac{<f,\phi_i>}{<\phi_i,\phi_i>}=\frac{\int_a^b f(x)\phi_i(x)dx}{\int_a^b \phi_i(x)\phi_i(x)dx}\ ; \quad <\phi_i,\phi_j>=0 \ \text{ if i} \neq \text{j}.$$

Show that this series expansion is equivalent to applying the least square method to approximate the function in terms of the orthogonal functions ϕ_i's. (Hint: Define the squared error norm as the integral of the squared error function.) (10 Points)