Elementary Numerical Analysis

2008 년 2 학기

Midterm Examination 2

Nov. 6, 2008

1. Give the following: (15 Points)

   a. A rough formula for the number of operations needed for the Gauss elimination process in terms of the rank of the matrix *n* and then the increase factor in the computational burden for the Gauss elimination solution of a linear system originating from the discretization of a 3D particle diffusion equation when the mesh size is refined to enhance the solution accuracy by *one order of magnitude*.

   b. Three important aspects of the iterative method as opposed to the direct method in the *practical* solution of the large sparse linear systems arising in numerous engineering problems

   c. Formula to determine the dominance ratio during the power iteration.

2. Explain the following with supporting equations: (15 Points)

   a. Spectral radius of the iteration matrix and its significance with respect to convergence.
   b. The fact that the power method converges even with scaling with the estimated eigenvalues.
   c. The decontamination method is in general more efficient than the deflation method, but it has its own limitation.

3. Show the following rigorously: (15 Points)

   a. Jacobi iteration converges unconditionally if the coefficient matrix is diagonally dominant.
   b. ~~The power method converges even with scaling with the estimated eigenvalue.~~
   c. The SOR iteration matrix is $T = -(D + \omega L)^{-1}(\omega U - (1-\omega)D)$. Start the derivation from the pointwise extrapolation scheme using the Gauss-Seidel estimate and the previous iterate. If you can't, give an alternative explanation.

4. Explain the following regarding the single parameter Chebyshev acceleration (15 Points)

   a. The extrapolation of the power method using the iteration dependent parameters yields the following expression for the eigenvector estimate:

$$x^{(k)} = c_1 \eta_k(\gamma_1) u_1 + \sum_{i=2}^{n} c_i \eta_k(\gamma_i) u_i$$

   where

$$\eta_k(\gamma_i) = \prod_{p=1}^{k} \left(\omega^{(p)} \sigma \frac{\gamma_i + 1}{2} + \left(1 - \omega^{(p)}\right) \right)$$

$$\gamma_i = 2\frac{\lambda_i}{\lambda_2} - 1 \text{ with } \lambda_n << 1$$

   b. The rationale that $\eta_k(\gamma_i)$ should be the Chebyshev polynomial of order *k*.

   c. The procedure to determine $\omega^{(p)}$ for given *k*.

Elementary Numerical Analysis

2008 년 2 학기

5. You want to find all the eigenvectors of the following matrix using various methods. Answer the following questions regarding the power method and the Q-R method. (40 Points)

$$A = \begin{bmatrix} 5 & -3 & -4 \\ -3 & 5 & 0 \\ -4 & 0 & 10 \end{bmatrix}$$

a. Apply the power method to find the largest eigenvalue for the first two steps starting from the initial guesses for the eigenvalue and the eigenvector of 1.0 and $[1,1,1]^T$, respectively.

b. Explain the inverse power method to find the smallest eigenvalue respectively. Do not apply the inverse of the matrix directly. Suppose that the L and U factors of A are known.

c. Apply the Householder algorithm to transform A to a Hessenberg form.

d. Find the first component of the R matrix, namely, r_{11}, needed in the Q-R factorization using the plane rotation procedure. You don't have to find the corresponding normalized vector.

e. Explain the rest of the Q-R algorithm needed to be applied to determine all the eigenvalues and eigenvectors.