Elementary Numerical Analysis

2008 년 2 학기

Final Examination

Dec. 6, 2008

1. Explain the following. (20 points)
   a. Simpson's method of integration. Include the formula in the answer.
   b. Modified linear interpolation method for finding roots.
   c. Implicit Euler method and its advantages and disadvantages when compared with the explicit method.
   d. Adams-Bashforth-Moulton method and its practical implementation.
   e. Multistep method and its advantages and disadvantages when compared with the multipoint method

2. Consider the following matrix eigenvalue equation resulting from the discretization of a multidimensional boundary value problem: $Ax = \lambda Sx$. The **minimum** eigenvalue λ is sought. Answer the following. (20 points)
   a. Derive the regular power iteration algorithm involving an inverse of a matrix with a formula to update the eigenvalue. *Be careful about that the form of the matrix eigenvalue problem is slightly different from what you had in HW 7.*
   b. Convert it to the **inverse** power method form and justify the need for this conversion.
   c. Define the dominance ratio and derive the practical method to estimate the dominance ratio during the iteration.
   d. Explain how the dominance ratio should be used to avoid the false convergence.
   e. Discuss about the possibility of having a wrong estimate of the dominance ratio by the formula you derived for problem *c*.

3. Consider a 3rd order polynomial to be used for numerical integration and also for solution of first order differential equations. Suppose four function values f_0, f_1, f_2, f_3 given with constant meshing (h), derive the following. (15 points)
   a. weighting factor for the first function value (f_0) in the integral from 0 and 3h.
   b. weighting factor for f_0 in the increment of the solution of $\frac{dy}{dt} = f(t, y)$, at the last interval (the 3rd interval). Suppose that y_0, y_1, and y_2 are known and you are to find y_3 using a third-order closed multipoint method.
   c. weighting factor for the first function value (f_0) in the increment in case of Adams-Bashforth method.

4. Answer the following regarding the Gauss quadrature method. (15 points)
   a. Explain the basic idea of the Gauss Quadrature method for numerical integration with an example of two integration points.
   b. Show that the roots of Legendre polynomial of order *n* become the integration points for integration with the precision order of 2*n*-1.

5. Answer the following regarding the Newton-Raphson method. (15 points)
   a. Give the algorithm in a pseudo-code form assuming that the lower and upper bounds of the interval within which the root resides is known.
   b. Show that the Newton-Raphson method is a unconditionally-convergent fixed point iteration scheme.
   c. Suppose that there are multiple roots in a specified interval and you are to find all the roots. You can achieve the goal without modifying the algorithm you setup for finding a **single** root by slightly modifying the function itself if you use the previously found roots wisely.

How can you do that? Hint: $f(x) = c\prod_{i=1}^{n}(x - \xi_i)$ where ξ_i's are the roots of $f(x)$. Think about a way of transforming the function such that it won't have the previously-found roots, say, $\xi_i, i = 1 \cdots k, (k < n)$.

6. Answer the following regarding the Runge-Kutta method. (15 points)
   a. Give the algorithm to implement the Runge-Kutta method.
   b. One of the relations to be used for determining $\alpha_i's$ and $\beta_i's$ is $\omega_2\alpha_1^2 + \omega_3\alpha_2^2 + \omega_4\alpha_3^2 = \frac{1}{3}$.
      Derive this relation using the coefficient match condition for $f_{xy}f$ which comes from the third order term. Use the fact $\alpha_i = \beta_i$. If you can't, use the condition for f_{xx}.
   c. The Runge-Kutta method can be used for solving higher order differential equations although it is derived for the 1-st order differential equation. How can you do that?

7. Among the topics you've learned from this course for the whole semester, give three topics or points you think important and worthwhile to recommend for your juniors to learn. Provide a little bit of reasons for your choice. (Bonus points)

Congratulations! You've made through a tough course!