Midterm Exam Solution (2008.10.21. Tue)

1. (a) Conservation of Angular momentum 5
  15 10
  " Energy (kinetic + Potential) 5

(b) i. In circular coordinate. Particle have both
10
<u>angular velocity</u> and <u>radial velocity</u>
5 5

$$a_c = \underline{2\dot{r}\dot{\theta}}\,\bar{e}_\theta$$

ii.
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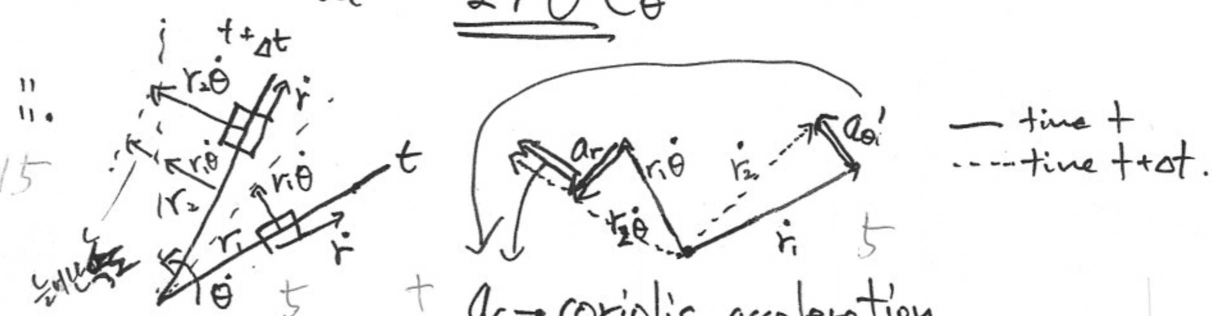

5 + 5
<u>a_c → coriolis acceleration</u> //

$a_c = 2\dot{r}\dot{\theta}$ → rotation velocity and radial velocit make corialis acceleration. ~~Because increasing of radius cause increasing of tangential velocity in constant angular velocity ($\dot{\theta}$).~~ 5 ⇒ • rate of chage of direction of vector V
• rate of chang of amount of vector V

iii. $\ddot{\theta} = 0$ $\dot{\theta} = \omega$ → const
10
To maintain a constant angular speed ω
We have to apply the torque $(\bar{r} \times \underbrace{2\dot{r}\dot{\theta}\,\bar{e}_\theta}_{\text{corialis term}}) \cdot M$
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(c) i. $e = \dfrac{\int R\,dt}{\int P\,dt}$ (Ratio of magnitude of the impulse corresponding respectively to the period of of restitution and to the period of deformation)
10 10

OR (relative velocity ratio of Before and After impact)

ii. conservation of linear momentum. 10
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계산과정 실수시

최초실수에 한하여 -2

2. (a)

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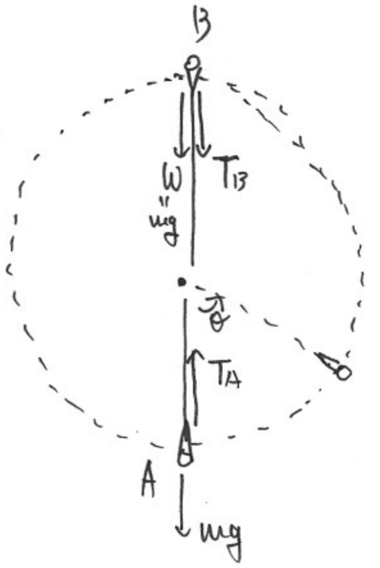

FBD 5

$\downarrow g.$

- Force Equation

state A : $\downarrow r$, $\rightarrow \theta$ (r-direction) $\quad -T_A + mg = m(\ddot{r} - r\dot{\theta}_A^2)$ --- ①

state B : $\uparrow r$, $\rightarrow \theta$ (r-direction.) $\quad -T_B - mg = m(\ddot{r} - r\dot{\theta}_B^2)$ ---- ②

· condition : $\ddot{r} = 0$, $2T_B = T_A$

① → $-T_A + mg = -mr\dot{\theta}_A^2$ --- ①'

② → $-\frac{T_A}{2} - mg = -mr\dot{\theta}_B^2$ --- ②'

→ 5

- Energy Conservation

$$\frac{1}{2}mr^2\dot{\theta}_B^2 + mg(2r) = \frac{1}{2}mr^2\dot{\theta}_A^2 + 0$$

$$E_{K_B} + E_{P_B} = E_{K_A} + E_{P_A}$$

$$\Rightarrow \dot{\theta}_B^2 = \dot{\theta}_A^2 - \frac{4g}{r} \quad \text{---- ③}$$

→ 5

①', ②' → $mr\dot{\theta}_A^2 + mg + 2mg = 2mr\dot{\theta}_B^2$

$$\dot{\theta}_A^2 + \frac{3g}{r} = 2\dot{\theta}_B^2 \quad \text{--- ④}$$

③, ④

$$\dot{\theta}_A^2 + \frac{3g}{r} = 2\dot{\theta}_A^2 - \frac{8g}{r}$$

$$\dot{\theta}_A^2 = \frac{11g}{r}$$

$$\dot{\theta}_A = \sqrt{\frac{11g}{r}} \;//, \quad v_A = r \cdot \dot{\theta}_A = \underline{\underline{\sqrt{11rg}}} \;//$$

$$③ \rightarrow T_A = mg + mr\dot{\theta}_A^2$$

$$= mg + 11mg = \underline{\underline{12mg}} \;//$$

(b) i)

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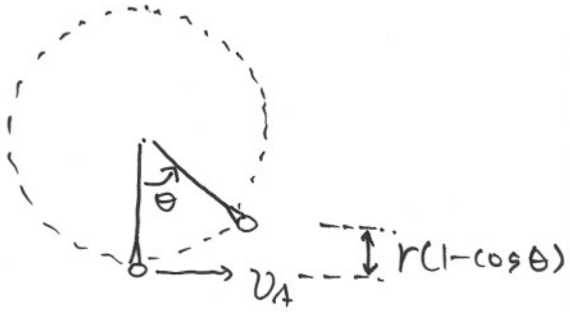

∘ Energy Conservation

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_\theta^2 + mgr(1-\cos\theta)$$

$$v_\theta^2 = v_A^2 - 2gr(1-\cos\theta)$$

$$\underline{\underline{v_\theta = \sqrt{v_A^2 - 2gr(1-\cos\theta)}}} \;//$$

→10

ii) FBD at θ

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→ 5

force (r-direction) : $-T + mg\cos\theta = -m\frac{v_\theta^2}{r}$

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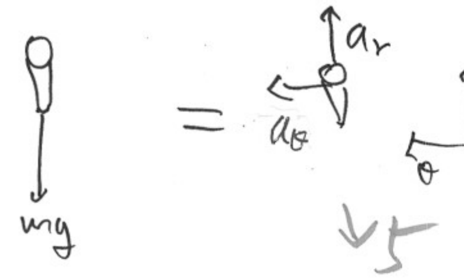
$$T = mg\cos\theta + \frac{m(v_A^2 - 2gr(1-\cos\theta))}{r} \;//$$

iii) At B. enough speed required

10 If. $mg > m\frac{v_B^2}{r}$, we can't maintain circular motion



$$r : -mg = -m\frac{v_B^2}{r} \quad (T=0 \rightarrow \text{minimum})$$

circular motion

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$$\Rightarrow \quad mg \leq m\frac{v_B^2}{r} \qquad (v_B^2 = v_A^2 - 4gr \ (③ \times r^2))$$

$$g \leq \frac{v_A^2 - 4gr}{r}$$

$$v_A^2 \geq rg + 4gr$$

$$\therefore \quad v_A \geq \sqrt{rg + 4rg}$$

$$v_A \geq \underline{\underline{\sqrt{5rg}}} \;//$$

minimum v_A required

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iv) $T = mg\cos\theta + \frac{m}{r}(v_A^2 - 2gr(1-\cos\theta))$ (ii)

10

$$\frac{dT}{d\theta} = -mg\sin\theta - 2mg\sin\theta = -3mg\sin\theta \rightarrow 5$$

T_{max} at $\frac{dT}{d\theta} = 0$,

$$-3mg\sin\theta = 0 \qquad \theta = n\pi.$$

① $n = 0$: $\theta = 0$ if $\theta < 0$ $\left.\frac{dT}{d\theta}\right|_{\theta<0} \rightarrow +$

$\theta > 0$ $\left.\frac{dT}{d\theta}\right|_{\theta>0} \rightarrow -$ $+ \; 0 \; -$ maximum point

5

$\therefore$ max $\theta = 0$ $T_{max} = mg + m\frac{v_A^2}{r} \;//$

각 5점씩

3. (a) ① Impact period is short enough

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→ if not : gravity force affect the impact force

② no friction

→ if not : After impact, Ball ratates

(b)

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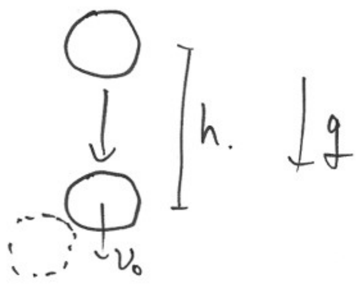

Energy conservation

$$\frac{1}{2}mv_0^2 = mgh$$

$$v_0 = \sqrt{2gh}$$ //

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(c)

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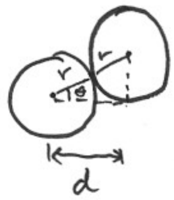
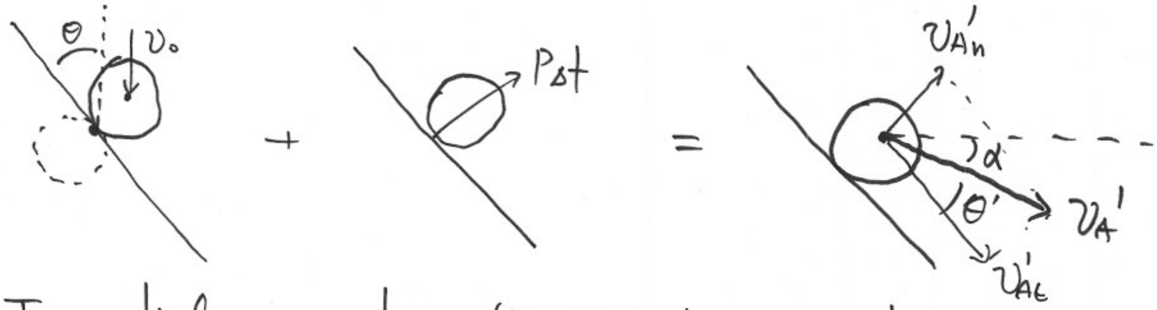


$\cos\theta = \frac{d}{2r}$ $\qquad \theta = \cos^{-1}\frac{d}{2r}$

Ball A

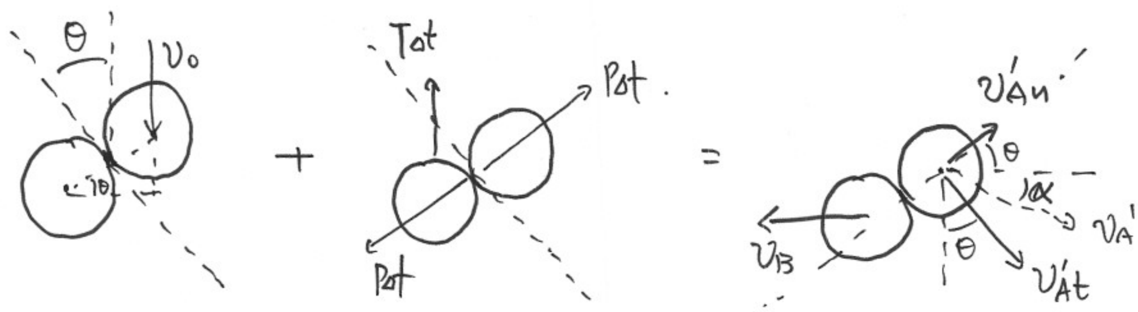


→ 5

∘ Tangential momentum (Ball A) conservation.

$$v_0\cos\theta = v'_{At} \quad \cdots\cdots ①.$$

Ball A, B



∘ x-direction momentum (A, B) conservation

$$0 + 0 = -mv_B + mv'_A\cos\alpha$$

$$0 = -mv_B + m(v'_{An}\cos\theta + v'_{At}\sin\theta) \cdots\cdots ②$$

→5

∘ Relative velocity on the line of impact

$$v'_{Bn} - v'_{An} = e(v_{An} - \cancel{v_{Bn}}^{\,0})$$

$$-v_B\cos\theta - v'_{An} = -e\,v_0\sin\theta \;------\; ③$$

→5

$$①,② \rightarrow \quad v_B = v'_{An}\cos\theta + v_0\cos\theta\sin\theta \;\cdots\; ④$$

$$③,④ \rightarrow \quad v_B = -v_B\cos^2\theta + e\,v_0\cos\theta\sin\theta + v_0\cos\theta\sin\theta$$

$$(1+\cos^2\theta)\,v_B = (1+e)\,v_0\cos\theta\sin\theta$$

$$\cancel{v_B = (1-e)v_0\frac{\cos\theta}{\sin\theta} = (1-e)v_0\cot\theta}$$

$$\underline{\underline{v_B}} = \frac{(1+e)v_0\cos\theta\sin\theta}{1+\cos^2\theta} \;---\; ⑤$$

//

③.⑤ → $\underline{\underline{v'_{An}}} = e v_0 \sin\theta - \dfrac{(1+e) v_0 \cos^2\theta \sin\theta}{1+\cos^2\theta}$ //

$\underline{\underline{v'_{At}}} = v_0 \cos\theta$ //

$$\left.\begin{aligned} \theta' &= \tan^{-1}\left(\frac{v'_{An}}{v'_{At}}\right) \\ v'_A &= \sqrt{{v'_{At}}^2 + {v'_{An}}^2} \end{aligned}\right\}$$ e=1 이 아니라서 복잡해짐.

(d) Energy loss ($E_{Before} - E_{after}$)

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$$= \underline{\frac{1}{2} m (v_0)^2 - \left(\frac{1}{2} m {v'_B}^2 + \frac{1}{2} m {v'_A}^2\right)}$$ //

→10

(e) $mgh = \frac{1}{2} m {v'_B}^2$

5

$$= \frac{1}{2} m \left(\frac{(1+e) v_0 \cos\theta \sin\theta}{1+\cos^2\theta}\right)^2$$

$$h = \frac{1}{2g}\left(\frac{(1+e) v_0 \cos\theta \sin\theta}{(1+\cos^2\theta)}\right)^2$$

→5

(f) $\dfrac{dh}{d\theta} = 0$, maximum point : θ_m. 5

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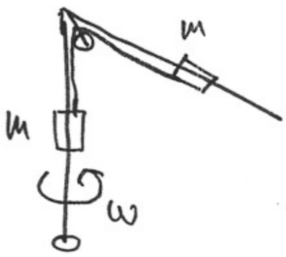
Check. $\left.\dfrac{dh}{d\theta}\right|_{\theta_m} = 0$

$\theta < \theta_m$	$\theta = \theta_m$	$\theta > \theta_m$
+	0	−

→5

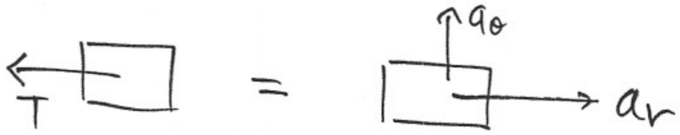
maximum point.

4. 50

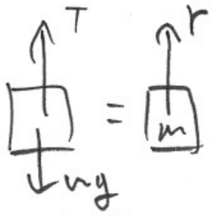


FBD (top view)

collar A



$T = mg$ +5



(a) ~~$T = mg \to$ fix~~ 15

~~Force (r-direction) of A~~

~~$-T = m(\ddot{r} - r\dot{\theta}^2)$~~

~~$-mg = m(\ddot{r} - r\dot{\theta}^2)$~~

~~$\ddot{r} = r\omega^2 - g$~~

$\dot{\theta}$

5
$$\begin{cases} -mg + T = m\ddot{r} \\ -T = m(\ddot{r} - r\dot{\theta}^2) \end{cases}$$

5

$-mg = 2m\ddot{r} - mr\dot{\theta}^2$

$$\ddot{r} = \frac{r\omega^2 - g}{2}.$$

$$T = m\frac{r\omega^2 + g}{2}.$$

(b) Not to move : $\ddot{r} = 0$ 5

$-mg = -mr_0\dot{\theta}^2$

$\dot{\theta}^2 = \frac{g}{r_0} \quad \dot{\theta} = \omega = \sqrt{\frac{g}{r_0}}$

5

(c) $\dot{\theta} = \omega > \sqrt{\frac{g}{r}}$

15

Transeverse velocity at $2r_0$ 5

$$v_\theta = 2r_0 \cdot \dot{\theta} = \underline{\underline{2r_0\omega}}$$

Transeverse Acceleration at $2r_0$ ($\dot{\theta} = \omega$ (const), $\ddot{\theta} = 0$)

$$a_\theta = \cancel{r\ddot{\theta}}^{\to 0} + 2\dot{r}\dot{\theta}$$
$$= 2\dot{r}\dot{\theta}$$ 5

To Find $\dot{r}$

$$\ddot{r} = r\dot{\theta}^2 - g \quad \underline{\text{ans(a)}}$$

$$\frac{d\dot{r}}{dt} = \frac{d\dot{r}}{dr}\frac{dr}{dt} = \underline{\dot{r}\frac{d\dot{r}}{dr} = r\dot{\theta}^2 - g.}$$

$$\dot{r}\,d\dot{r} = (\underline{r\dot{\theta}^2 - g})\,dr$$ 5

$$\left[\frac{1}{2}\dot{r}^2\right]_0^{\dot{r}} = \left[\frac{1}{4}r^2\dot{\theta}^2 - \frac{gr}{2}\right]_{r_0}^{2r_0}$$

$$\frac{1}{2}\dot{r}^2 = \frac{1}{4}(4r_0^2 - r_0^2)\dot{\theta}^2 - \frac{1}{2}g(2r_0 - r_0)$$

$$\dot{r}^2 = \frac{3}{2}r_0^2\dot{\theta} - \cancel{2}g r_0$$

$$\dot{r} = \sqrt{\frac{3}{2}r_0^2\dot{\theta} - \cancel{2}g r_0}$$

$$\dot{\theta} = \omega.$$

$$\therefore \underline{\underline{a_\theta = 2\cdot\sqrt{\frac{3}{2}r_0^2\omega - \cancel{2}g r_0}\cdot\omega}}$$

$$\qquad\qquad\qquad T \qquad\qquad\qquad\qquad P$$

(d) $E_{r_0} = \frac{1}{2} m r_0^2 \omega^2 \quad 5 \quad + \quad 0$

1.5

$E_{2r_0} = \frac{1}{2} m (2r_0^2)\omega^2 \quad 5 \quad + \underline{mgr_0} + \frac{1}{2} m \dot{r}^2$

collar B $\quad \left(\frac{1}{2} m \left(\frac{3}{2} r_0^2 \dot{\theta} - \cancel{4} g r_0\right)\right)$

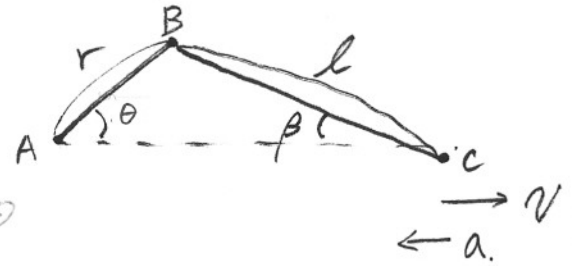
$$E_{2r_0} - E_{r_0} = 2 m r_0^2 \omega^2 - \frac{1}{2} m r_0^2 \omega^2 = \underline{\underline{\frac{3}{2} m r_0^2 \omega^2}} \text{ (increas}$$

5

$\underline{\underline{+ mgr_0}}$

$\underline{\underline{+ \frac{1}{2} m \left(\frac{3}{2} r_0^2 \dot{\theta} - \cancel{4} g r_0\right)}}$

5.



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(a) $\omega_{BC} = ?$

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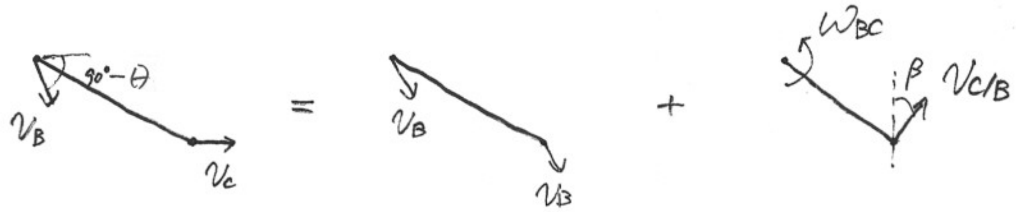

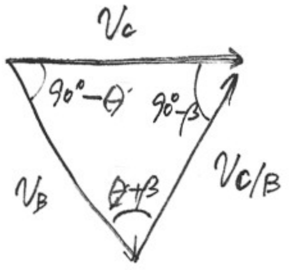

$$v_{C/B} = l\,\omega_{BC}$$

$$\frac{v_C}{\sin(\theta+\beta)} = \frac{v_{C/B}}{\sin(90^\circ-\theta)} = \frac{v_B}{\sin(90^\circ-\beta)}$$

→ 5

$$v_{C/B} = \frac{v\sin(90^\circ-\theta)}{\sin(\theta+\beta)} \qquad \omega_{BC} = \frac{v\sin(90^\circ-\theta)}{l\sin(\theta+\beta)} = \frac{v\cos\theta}{l\sin(\theta+\beta)}$$

→ 5

(b) $\omega_{AB} = ?$

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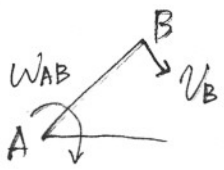

$$v_B = \frac{v\sin(90^\circ-\beta)}{\sin(\theta+\beta)} = \frac{v\cos\beta}{\sin(\theta+\beta)}$$

$$v_B = r\,\omega_{AB}$$

→ 5

$$\omega_{AB} = \frac{v\cos\beta}{r\sin(\theta+\beta)}$$

→ 5

(c) $\theta = 60^\circ$ $\quad l = \sqrt{3}\,r$

$\beta = 30^\circ$

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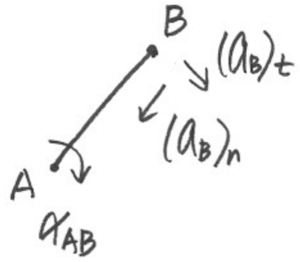
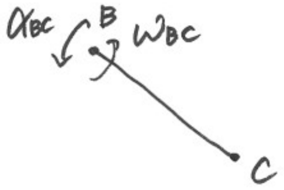


$(a_B)_n = r\,\omega_{AB}^2$

$(a_B)_t = r\,\alpha_{AB}$

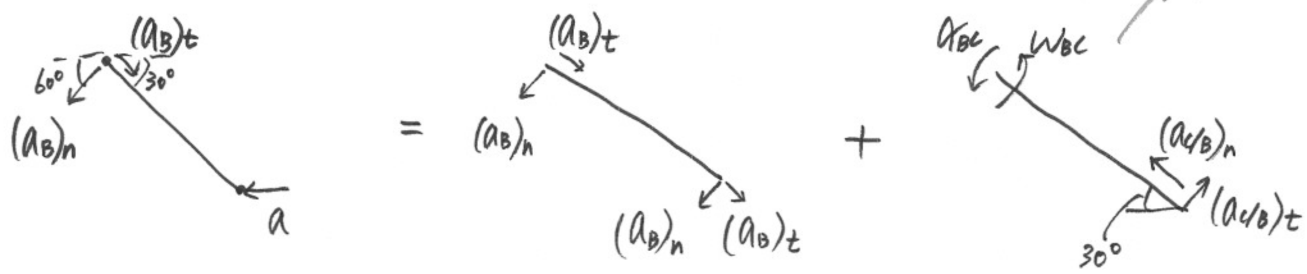


$(a_{C/B})_n = l\,\omega_{BC}^2$

$(a_{C/B})_t = l\,\alpha_{BC}$

→ 5

→ 5



$$a = (a_B)_n + (a_B)_t + (a_{C/B})_n + (a_{C/B})_t$$

$\overset{+}{\rightarrow} x$

$$-a = -(a_B)_n\cos\theta + (a_B)_t\cos(90^\circ - \theta) - (a_{C/B})_n\cos\beta + (a_{C/B})_t\sin\beta$$

5

$$-a = -r\omega_{AB}^2\cos\theta + r\alpha_{AB}\sin\theta - l\omega_{BC}^2\cos\beta + l\alpha_{BC}\sin\beta \quad ①$$

5 $+\uparrow y$

$$0 = -(a_B)_n\sin\theta - (a_B)_t\sin(90^\circ - \theta) + (a_{C/B})_n\sin\beta + (a_{C/B})_t\cos\beta$$

$$0 = -r\omega_{AB}^2\sin\theta - r\alpha_{AB}\cos\theta + l\omega_{BC}^2\sin\beta + l\alpha_{BC}\cos\beta \quad ②$$

$\theta = 60^\circ \quad l = \sqrt{3}r \quad \beta = 30^\circ$

$$\omega_{BC} = \frac{v\cos\theta}{l\sin(\theta+\beta)} = \frac{v\cos 60^\circ}{\sqrt{3}r} = \frac{v}{2\sqrt{3}r}$$

$$\omega_{AB} = \frac{v\cos\beta}{r\sin(\theta+\beta)} = \frac{\sqrt{3}v}{2r}$$

① & ②에 대입 (plug in)

①》

$$-a = -\cancel{r}\frac{3v^2}{4r^{\cancel{2}}} \times \frac{1}{2} + r\alpha_{AB}\frac{\sqrt{3}}{2} - \sqrt{3}\cancel{r}\frac{v^2}{4\,12r^{\cancel{2}}}\frac{\sqrt{3}}{2} + \sqrt{3}r \times \frac{1}{2}\alpha_{BC}$$

$$-a = -\frac{3v^2}{8r} + \frac{\sqrt{3}r}{2}\alpha_{AB} - \frac{v^2}{8r} + \frac{\sqrt{3}}{2}r\alpha_{BC}$$

$$\frac{\sqrt{3}}{2}r(\alpha_{AB} + \alpha_{BC}) = \frac{v^2}{2r} - a \quad — ③$$

②》

$$0 = -\cancel{r}\frac{3v^2}{4r^{\cancel{2}}}\frac{\sqrt{3}}{2} - r\alpha_{AB}\frac{1}{2} + \sqrt{3}\cancel{r}\frac{v^2}{12r^{\cancel{2}}}\frac{1}{2} + \sqrt{3}r\frac{\sqrt{3}}{2}\alpha_{BC}$$

$$0 = -\frac{3\sqrt{3}v^2}{8r} - \frac{r}{2}\alpha_{AB} + \frac{\sqrt{3}v^2}{24r} + \frac{3}{2}r\alpha_{BC}$$

24

$$\frac{r}{2}\alpha_{AB} = -\frac{\sqrt{3}v^2}{3r} + \frac{3}{2}r\alpha_{BC} \quad — ④$$

③ & ④

$$\sqrt{3}\left(-\frac{\sqrt{3}v^2}{3r} + \underbrace{\frac{3}{2}r a_{BC} + \frac{r}{2}a_{BC}}_{2r a_{BC}}\right) = \frac{v^2}{2r} - a$$

$$2r a_{BC} = \frac{v^2}{2\sqrt{3}r} - \frac{a}{\sqrt{3}} + \frac{\sqrt{3}v^2}{3r}$$

$$2r a_{BC} = \frac{\sqrt{3}v^2}{2r} - \frac{a}{\sqrt{3}}$$

$$\therefore a_{BC} = \frac{\sqrt{3}v^2}{4r^2} - \frac{a}{2\sqrt{3}r}$$

(d) $a_{AB} = ?$

10 where ④

$$\frac{r}{2}a_{AB} = -\frac{\sqrt{3}v^2}{3r} + \frac{3}{2}r a_{BC}$$

$$\frac{r}{2}a_{AB} = -\frac{\sqrt{3}v^2}{3r} + \frac{3}{2}\cancel{r}\left(\frac{\sqrt{3}v^2}{4r^{\cancel{2}}} - \frac{a}{2\sqrt{3}\cancel{r}}\right)$$

$$\frac{r}{2}a_{AB} = -\frac{\sqrt{3}v^2}{3r} + \frac{3\sqrt{3}v^2}{8r} - \frac{\sqrt{3}a}{4r}$$

$$\therefore a_{AB} = \frac{\sqrt{3}v^2}{12r^2} - \frac{\sqrt{3}a}{2r}$$

(C), (d) 문제는 연결되는 식으로 풀어야 함으로

점수를 최초식을 세우는 것에 배분하였습니다.