final Solution.

1. (1). $F = ma = m\frac{dv}{dt}$

$$m\frac{dv}{dt} = m\frac{ds}{dt}\frac{dv}{ds} = mv\frac{dv}{ds}$$

$$F = mv\frac{dv}{ds}$$

~~Fdv~~ $F\,ds = mv\,dv$

$$\| = \int_{v_1}^{v_2} mv\,dv = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$U_{1\to2}$, $\frac{1}{2}mv_2^2 = T_2$, $\frac{1}{2}mv_1^2 = T_1$

(2) $F = ma$.

$$F = m\frac{dv}{dt}$$

$F\,dt = m\,dv \rightarrow \int F\,dt = mv_2 - mv_1$.

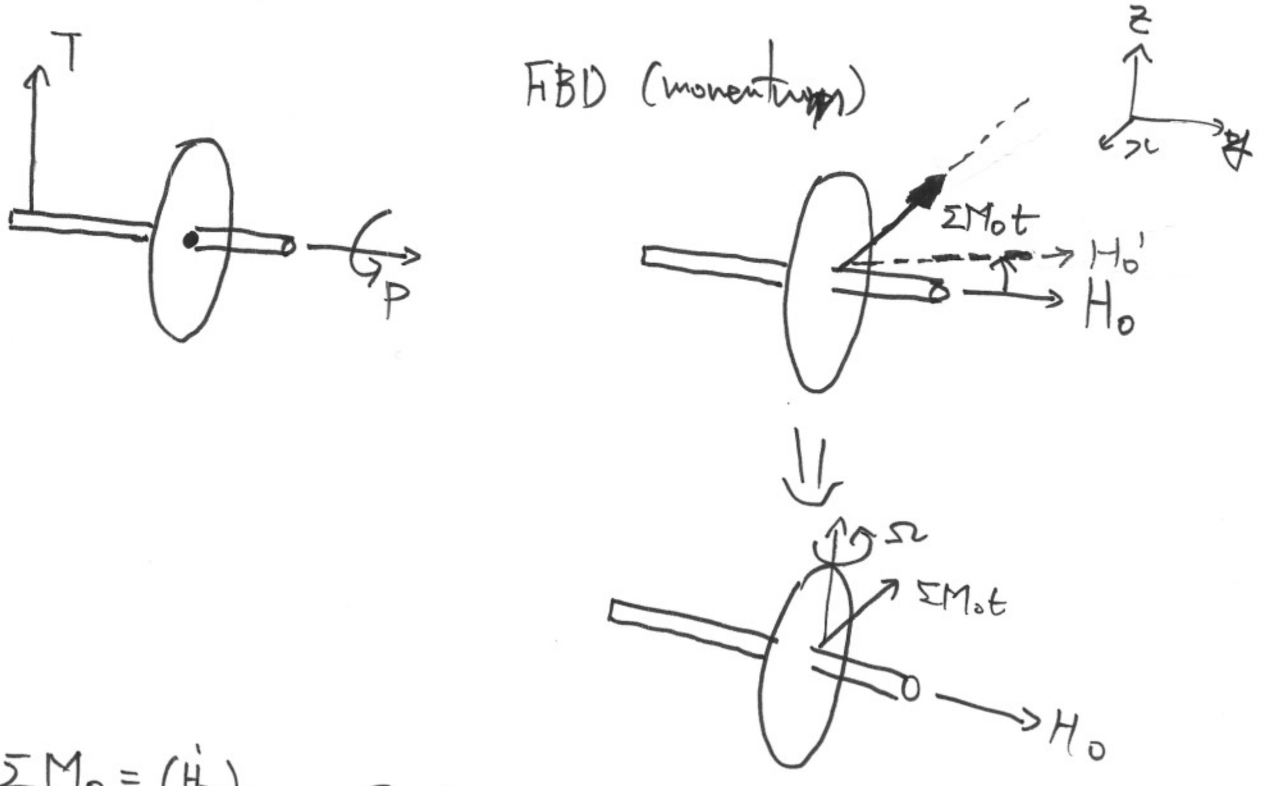
($\int F\,dt$ = impuls. mv = momentum)

$$mv_1 + \int_{t_1}^{t_2} F\,dt = mv_2$$ //

(3) i) The external forces acting on a rigid body are equivalent to the effective forces of the various particles forming the body.

ii) Dynamic problem ⟶ static problem

(4) i)



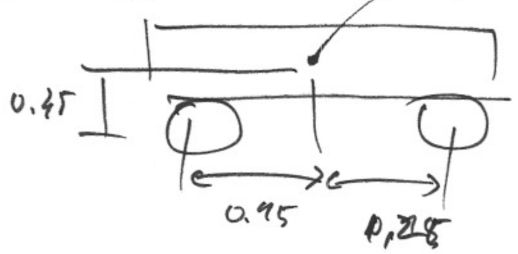
ii) $\Sigma M_o = (\dot{H}_o)_{Oxyz} + \Omega \times H_o$

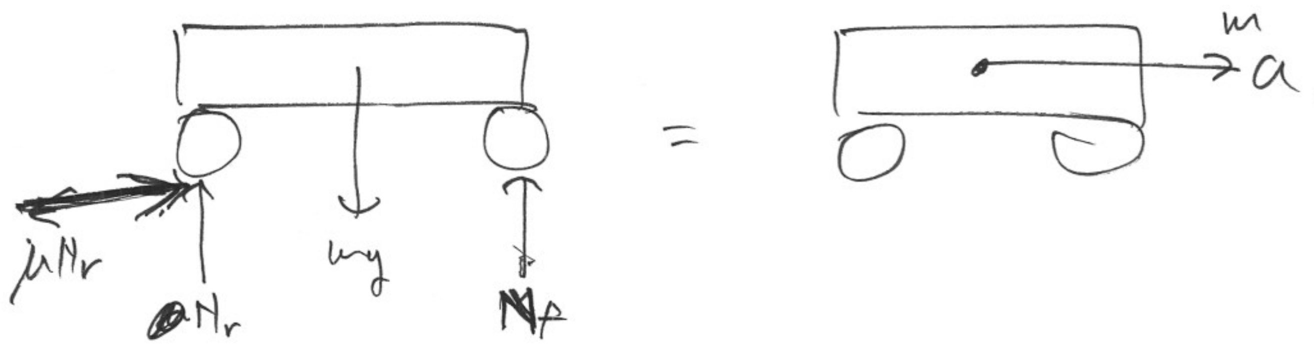
in precession $(\dot{H}_o)_{Oxyz} = 0$.

$\Sigma M_o = \Omega \times H_o$. //

Ω → +z 방향.

2. (a)





$\Sigma F_y = N_r + N_f - mg = 0.$

$N_r + N_f = 1500 \times 9.8$

$= 14700\,N.$ —— ①

$\Sigma F_x = \mu N_r = ma.$ —— ②

$\Sigma M_G^{(+)} = \mu N_r(0.35) + N_f(1.25) - N_r(0.95) = 0.$

$(\overset{0.2}{\mu}\,0.35 - 0.95)N_r + N_f(1.25) = 0.$ (0.07)

$-0.68\,N_r + 1.25\,N_f = 0.$ —— ③

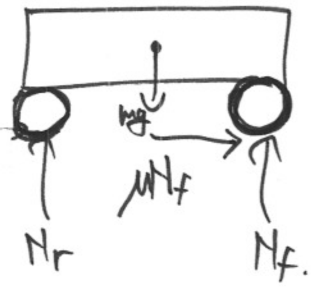
①, ③ $\longrightarrow N_r = 9520.7 \quad N_f = 5179.2.$

② $\rightarrow 0.2 \cdot 9520.7 = 1500 \times a.$ $\quad 100\ km/h = 27.78\ m/s.$

$a = 1.27\ m/s^2.$

$v = at \rightarrow 27.78 = 1.27 \times t \quad t \rightarrow \underline{\underline{21.87\ s}}$

(b) i) front



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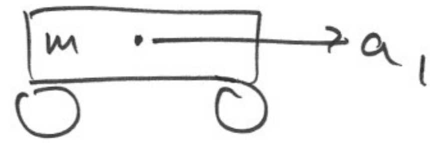
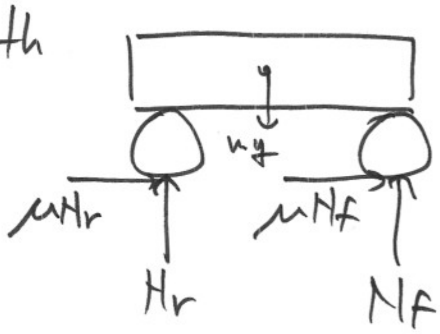


$$\mu N_{f_1} = m a_1$$

ii) both



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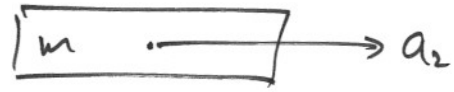
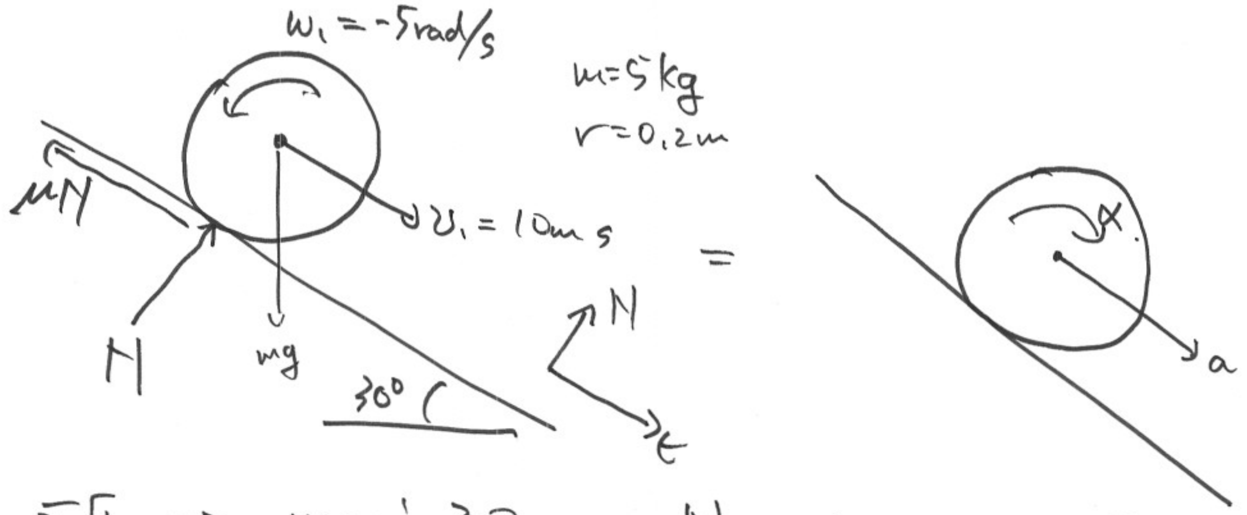


$$\mu (N_{r_2} + N_{f_2}) = m a_2.$$

$$N_{f_1} < N_{r_2} + N_{f_2} \qquad a_1 < a_2.$$

∴ when engine drives four wheels, less time is required.

3



$\Sigma F_t \Rightarrow \quad mg\sin 30 - \mu N = ma \quad —— ①$

$\Sigma F_n \Rightarrow \quad N - mg\sin 30 = 0$

$N = mg\cos 30° = 5 \cdot 9.8 \cdot \cos 30 = 42.44 N$

—— ②

$\Sigma M_G \Rightarrow (+\curvearrowright) \quad r \cdot \mu \cdot N = I\alpha$

$0{,}2 \cdot 0{,}3 \cdot 42{,}44 = \frac{2}{5} m r^2 \alpha \quad (r = 0.2)$

$\alpha = 31.83$ rad/s²

—— ③.

$① \rightarrow \quad 24.5 - 0{,}3 \cdot 42{,}44 = 5a$

$\underline{\underline{a = 2{,}3536 \text{ m/s}}}$

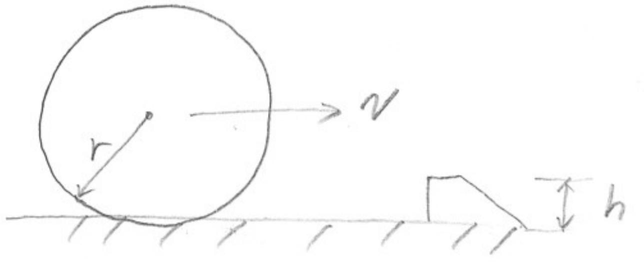
$\left. \begin{array}{l} v_2 = v_1 + at \\ \omega_2 = \omega_1 + \alpha t \end{array} \right)$

no slip $\Rightarrow \quad v_2 = r\omega_2$

$10 + 2.3536t = 0{,}2(-5 + 31.83t) \quad \underline{\underline{t = 2{,}741 \text{ sec}}}$

$d = v_1 t + \frac{1}{2}at^2 = 10 \cdot (2{,}741) + \frac{1}{2}(2.3536)(2{,}741)^2 = \underline{\underline{36{,}25 \text{ m}}}$

Problem 4 (20)



impact at A

$$I' = mk^2 + mr^2$$

$$mk^2\omega + mv(r-h) = I'\omega$$

$$= m(k^2+r^2)\,\omega$$

$$v' = v\left(1 - \frac{rh}{k^2+r^2}\right)$$

roll about point A
where ω will get from energy conservation method.
for minimum velocity

$$\frac{1}{2} I'\omega'^2 = mgh$$

$$\frac{1}{2} m(k^2+r^2)\,\omega'^2 = mgh$$

$$\omega'^2 = \frac{2gh}{k^2+r^2} \qquad \omega' = \sqrt{\frac{2gh}{k^2+r^2}}$$

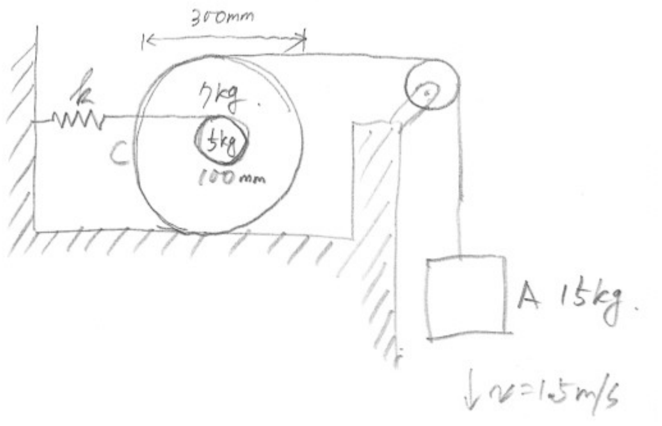
∴ minimum value of v

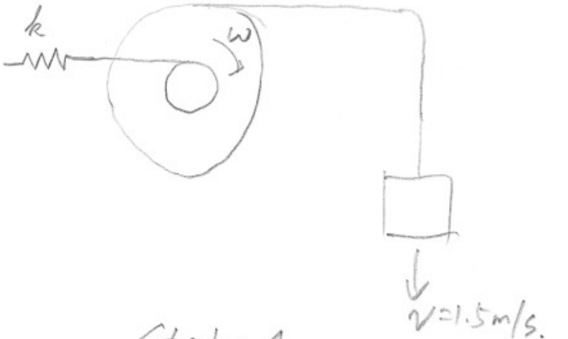
$$v = \frac{r}{k^2+r^2-rh}\sqrt{2gh(k^2+r^2)}$$

Problem 5 (40)

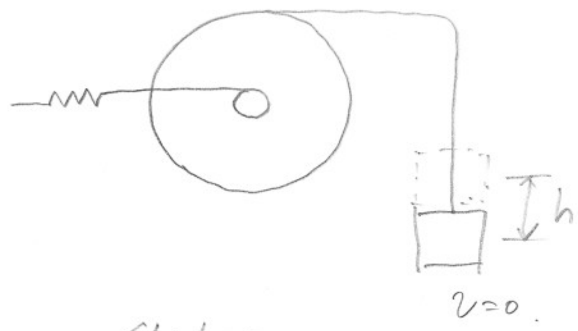
$k = 2000\ N/m$

spring stretch 100mm





State 1



State 2

$v = rw$

$v = 0.15 \times w$

$w = \frac{1.5}{0.15}$

$= 10\ rad/s$

$I_1 = 0.0625$

$I_2 = \frac{1}{2} m_2 r_2^2$

$T_1 = \frac{1}{2} k x_1^2 + \frac{1}{2} m v^2 + \frac{1}{2} I_1 w^2 + \frac{1}{2} I_2 w^2$

$V_1 = mgh = 0$

$T_2 = \frac{1}{2} k x_2^2$

$V_2 = -mgh$

(a) (10)

$m = \rho t A$

$m_1 = 17 \quad m_2 = 5$

$r_1 = 0.05 \quad r_2 = 0.15.$

$\frac{1}{2} M_2 r_2^2 - \frac{1}{2} m_1 r_1^2$

$= \frac{1}{2} \rho t (A_2 r_2^2 - A_1 r_1^2)$

$= \frac{1}{2} \frac{m}{A_2 - A_1} (A_2 r_2^2 - A_1 r_1^2)$

$I_2 = \frac{1}{2} m_2 (r_1^2 + r_2^2) \rightarrow 5$

$= 0.0625$

$I_1 = \frac{1}{2} m_1 r_1^2 = 0.00875 \rightarrow 2$

$I_c = I_1 + I_2 \rightarrow 3$

$= 0.07125$

$\frac{1}{2} \quad m_2 \frac{r_2^4 - r_1^4}{r_2^2 - r_1^2}$

$9r_1^2 \quad 10r_1^2$

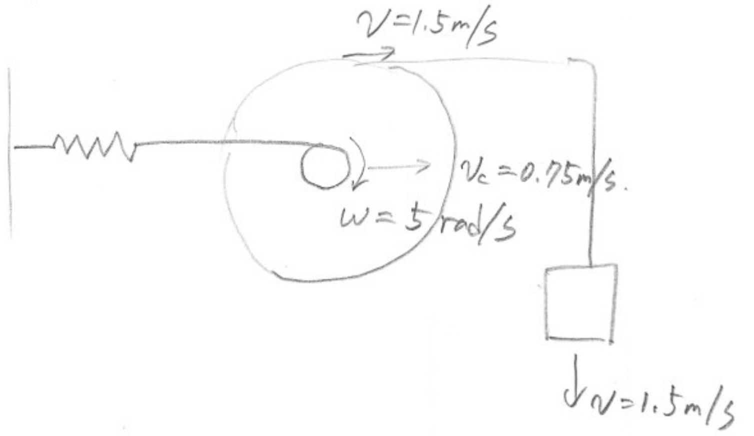
$I_2 = \frac{1}{2} m_2 (r_2^2 + r_1^2)$

$I_1 = \frac{1}{2} m_1 r_1^2.$

$I_c = I_1 + I_2$

$I_c = \frac{1}{2} m_2 10 r_1^2 + \frac{1}{2} m_1 r_1^2$

(b) 10



$m_c = 12$ kg

$v_c = 0.75$ m/s

$I_c = 0.07125$

$w = 5$ rad/s

$m_A = 15$ kg

$v = 1.5$ m/s.

$x_1 = 0.1$ m

$k = 2000$ N/m

$$v = rw$$

$$w = \frac{v}{r} = \frac{0.75}{0.15}$$

$$= 5 \text{ rad/s} \quad \rightarrow 2$$

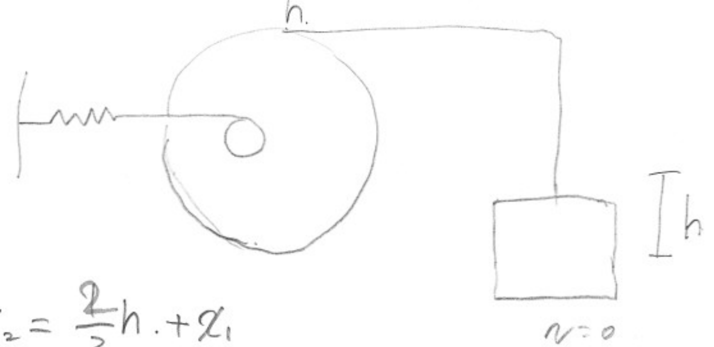
State 1 $v = 1.5$ m/s 일 때.

$$T_1 = \frac{1}{2} m_c v_c^2 + \frac{1}{2} I_c w^2 + \frac{1}{2} m_A v^2 = 21.14$$

(3.375, 0.89, 16.875)

$$V_1 = \frac{1}{2} k x_1^2 = 10 .$$

) 3



State 2

3 (

$$T_2 = 0$$

$$V_2 = -m_A g h + \frac{1}{2} k x_2^2 \quad x_2 = \frac{2}{3} h + x_1$$

$$= \frac{2}{3} h + 0.1$$

) → 3

$$\therefore \ T_1 + V_1 = T_2 + V_2$$

$$31.14 = 1000 \times \left(\frac{4h^2}{9} + \frac{1}{75} h + \frac{1}{100}\right) - 147.15h$$

$$111.11 h^2 - 80.48h - 21.14 = 0$$

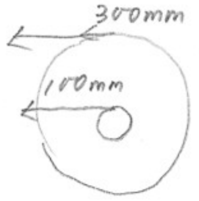
$$h = 0.93 \text{ m}$$

(c) 10 $T_3 = \frac{1}{2} I_c \omega_3^2 + \frac{1}{2} m_c v_{3c}^2 + \frac{1}{2} m_A v_A^2$

3 $V_3 = m_A g h_3$

when initial condition spring is stretched by 100 mm

So, $x_3 = 0$



$h_3 = 0.3\ m.$

$v_A = 2 v_{3c}$

$v_{3c} = r \omega_3$

$v_A = 2 r_2 \omega_3$

$v_{3c} = r_2 \omega_3$) → 2

$T_1 + V_1 = T_3 + V_3$

$T_3 = \frac{1}{2} I_c \omega_3^2 + \frac{1}{2} m_c r_2^2 \omega_3^2 + \frac{1}{2} m_A 4 r_2^2 \omega_3^2$

$= \left(\frac{1}{2} I_c + \frac{1}{2} m_c r_2^2 + 2 m_A r_2^2\right) \omega_3^2$

$= 0.8456\ \omega_3^2$

$V_3 = 44.145$

$\therefore 31.14 = 0.8456\ \omega_3^2 + 44.145$

$\omega_3 = 3.9$ rad/s

$v_A = 1.17$ m/s) → 5

(d)[10] Max distance

$T_4 = 0$

7($V_4 = \frac{1}{2}k x_4^2 + m_A g h_4 = \frac{1}{2}k\left(\frac{1}{9}h_4^2 - \frac{1}{15}h_4 + \frac{1}{100}\right) + m_A g h_4$

$= \frac{1000}{9}h_4^2 + (147.15 - 66.7)h_4 + 10$

$x_4 = \frac{2}{3}h_4 - x_1$

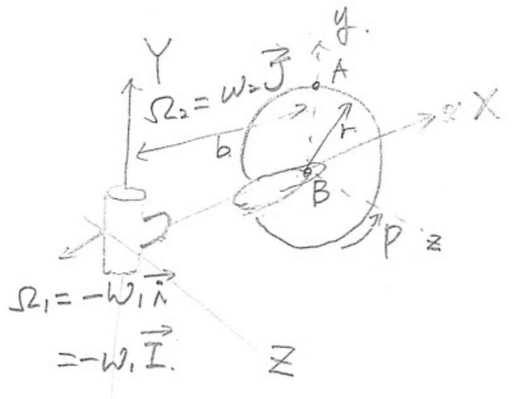
$= \frac{2}{3}h_4 - 0.1$) 3.

$T_1 + V_1 = T_4 + V_4$

$111.11 h_4^2 + 80.48 h_4 - 21.14 = 0$

$h_4 = 0.20$ m

Problem 6. (40)



$\Omega_2 = \omega_2 \vec{J} = \omega_2 \vec{j}$

$\Omega_1 = -\omega_1 \vec{i}$
$= -\omega_1 \vec{I}$

$\Omega = -\omega_1 \hat{i} + \omega_2 \hat{j}$

$\omega_{D/F} = p\vec{k}$

$I_{xx} = I_{yy} = \frac{1}{4}mr^2 \quad I_{zz} = \frac{1}{2}mr^2$ at point B.

14
(a) Kinetic Energy $T = ?$

5 $T = \frac{1}{2}mv^2 + \frac{1}{2}\omega \overline{H}_B$ $\quad \omega = -\omega_1\hat{i} + \omega_2\hat{j} + p\hat{k}$
$v = b\omega_2$

5 $\overline{H}_B = \hat{i} I_{xx}\omega_x + \hat{j} I_{yy}\omega_y + \hat{k} I_{zz}\omega_z$

4 $\omega_x = -\omega_1, \quad \omega_y = \omega_2, \quad \omega_z = p.$

$\therefore \overline{H}_B = -\frac{1}{4}mr^2\omega_1\hat{i} + \frac{1}{4}mr^2\omega_2\hat{j} + \frac{1}{2}mr^2p\hat{k}$

$$\therefore T = \frac{1}{2}mb^2\omega_2^2 + \frac{1}{2}(-\omega_1\hat{i} + \omega_2\hat{j} + p\hat{k})\cdot(-\frac{1}{4}mr^2\omega_1\hat{i} + \frac{1}{4}mr^2\omega_2\hat{j} + \frac{1}{2}mr^2p\hat{k})$$

$$= \frac{1}{2}mb^2\omega_2^2 + \frac{1}{8}mr^2\omega_1^2 + \frac{1}{8}mr^2\omega_2^2 + \frac{1}{4}mr^2p^2$$

$$= \frac{1}{2}mb^2\omega_2^2 + \frac{1}{8}mr^2(\omega_1^2 + \omega_2^2 + 2p^2)$$

$4b^2\omega$

$$= \frac{1}{8}mr^2\left[\omega_1^2 + \left(1 + \frac{4b^2}{r^2}\right)\omega_2^2 + 2p^2\right]$$

13

(b) 3 $\Omega = -\omega_1 \hat{i} + \omega_2 \hat{j}$ 2 $r = b\hat{i} + r\hat{j}$ 2 $\omega_{P/F} = p\hat{k}$

$r_{A/B} = r\hat{j}$

$v_A = v_{A'} + v_{A/F}$

3 $v_{A'} = \Omega \times r$

$= (-\omega_1 \hat{i} + \omega_2 \hat{j}) \times (b\hat{i} + r\hat{j})$

$= -r\omega_1 \hat{k} - b\omega_2 \hat{k}$

$= -(r\omega_1 + b\omega_2)\hat{k}$

3 $v_{A/F} = \omega_{P/F} \times r_{A/B} = p\hat{k} \times r\hat{j}$

$= -pr\hat{i}$

$\therefore v_A = -(r\omega_1 + b\omega_2)\hat{k} - pr\hat{i}$

$= -pr\hat{i} - (r\omega_1 + b\omega_2)\hat{k}$

(c) $\underline{a}_A = \underline{a}_B + \dot{\underline{\Omega}} \times \underline{r}_{A/B} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_B = -b\omega_2^2 \hat{i}$

$\dot{\underline{\Omega}} = -\omega_1 \hat{i} + \omega_2 \dot{\underline{J}} = -\omega_1 \underline{\Omega} \times \hat{i} = \omega_1 \omega_2 \hat{k}$

$\underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) = (-\omega_1 \hat{i} + \omega_2 \hat{J}) \times (-r\omega_1 \hat{k})$

$= -r\omega_1 (\omega_1 \hat{j} + \omega_2 \hat{i})$

$2\Omega \times v_{rel} = 2(-\omega_1 \hat{i} + \omega_2 J) \times (-rp\hat{i})$

$= 2rp\omega_2 \hat{k}$

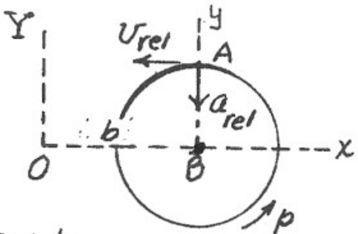
$a_{rel} = -rp^2 \hat{j}$

$\underline{\Omega} \times \underline{r}_{A/B} = \omega_1 \omega_2 \hat{k} \times r\hat{j} = -r\omega_1 \omega_2 \hat{i}$

$\therefore \underline{a} = -\omega_2 (b\omega_2 + 2r\omega_1)\hat{i} - r(\omega_1^2 + p^2)\hat{j} + 2rp\omega_2 \hat{k}$

7/46 Angular velocity of x-y-z axes is

$\underline{\Omega} = -\omega_1 \underline{i} + \omega_2 \underline{J}$



$\underline{v} = \underline{v}_A = \underline{v}_B + \underline{\Omega} \times \underline{r}_{A/B} + \underline{v}_{rel}$

where $\underline{v}_B = b\omega_2(-\underline{k}) = -b\omega_2\underline{k}$

$\underline{\Omega} \times \underline{r}_{A/B} = (-\omega_1 \underline{i} + \omega_2 \underline{J}) \times r\underline{j} = -r_1\omega_1 \underline{k}$

$\underline{v}_{rel} = -rp\underline{i}$

Thus $\underline{v} = -b\omega_2\underline{k} - r\omega_1\underline{k} - rp\underline{i} = \underline{-rp\underline{i} - (r\omega_1 + b\omega_2)\underline{k}}$

$\underline{a} = \underline{a}_A = \underline{a}_B + \dot{\underline{\Omega}} \times \underline{r}_{A/B} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

where $\underline{a}_B = -b\omega_2^2\underline{i}$

$\dot{\underline{\Omega}} = -\omega_1\dot{\underline{i}} + \omega_2\dot{\underline{J}} = -\omega_1\underline{\Omega}\times\underline{i} = \omega_1\omega_2\underline{k}$

$\underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) = (-\omega_1\underline{i} + \omega_2\underline{J}) \times (-r\omega_1\underline{k}) = -r\omega_1(\omega_1\underline{j} + \omega_2\underline{i})$

$2\underline{\Omega} \times \underline{v}_{rel} = 2(-\omega_1\underline{i} + \omega_2\underline{J}) \times (-rp\underline{i}) = 2rp\omega_2\underline{k}$

$\underline{a}_{rel} = -rp^2\underline{j}$, $\dot{\underline{\Omega}} \times \underline{r}_{A/B} = \omega_1\omega_2\underline{k} \times r\underline{j} = -r\omega_1\omega_2\underline{i}$

Substitute, combine & get

$\underline{a = -\omega_2(b\omega_2 + 2r\omega_1)\underline{i} - r(\omega_1^2 + p^2)\underline{j} + 2rp\omega_2\underline{k}}$