Structural Analysis – II

Fall Semester, 2006

- **Instructor & TAs**

  Instructor : 이 해 성

  Office : 38-404

  Tel : 880-8388/e-mail:chslee@snu.ac.kr/hompage:http://strana.snu.ac.kr

  TA : 류근원, 이승한 (38-427, 8740)

  Class hour : Mon. 16:00-16:50(17:00-17:50, Exercise)/Wed. 16:00-17:40

- **Text book, Reference and Programs**
  1. Class note (Main class material, will be posted at the web site)
  2. Structural Analysis - Classical and Matrix Approach, 2^{nd} Ed.
     by Jack C. McCormac and R. E. Elling (for reading and homework)
  3. Elementary Structural Analysis 4th Ed. by C. H. Norris
  4. Statically Indeterminate Structures by Chu-Kia Wang
  5. Computer Programs “SNUSEA” and “MDM” (available at the web site for free)

- **Class Contents**
  1. Brief Review of Flexibility Method with Influence Lines of Indeterminate Str. (1)
  2. Slope-Deflection Method (Stiffness Method) (2-5)
  3. Moment-Distribution Method (6-7)
  4. Energy Method (8-9)
  5. Matrix Method for Structural Analysis – Direct Stiffness Method (10-14)
  6. Buckling of Structures (14-15)

- **Evaluation**
  1. 2 Mid Term Exams : 40 % (Oct.16/Nov.13)
  2. Final Exams : 30 % (Dec. 13, 3-6^{PM})
  3. Home work and attendance : 30 %

- **Prerequisite class**
  - Mechanics of Materials
  - Structural Analysis I

- **No class:** October 4

* No cel-phone during class hour.

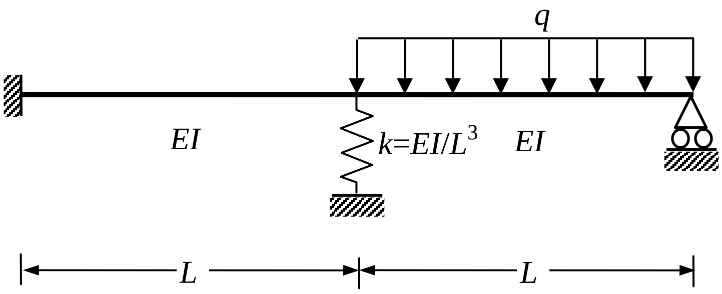
* All information presented here may be changed as needed by the instructor.

* Final grades are final. Absolutely no chance to alter them by any excuse.

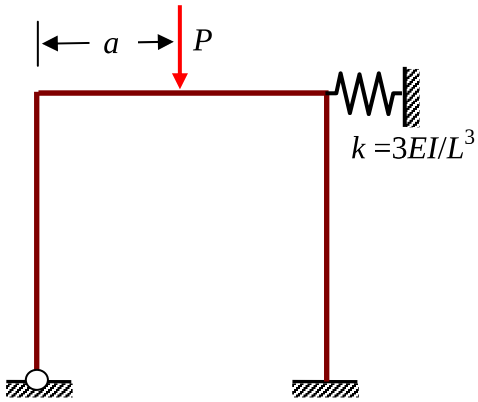
구조역학 II 중간고사 I

2006. 10. 16.

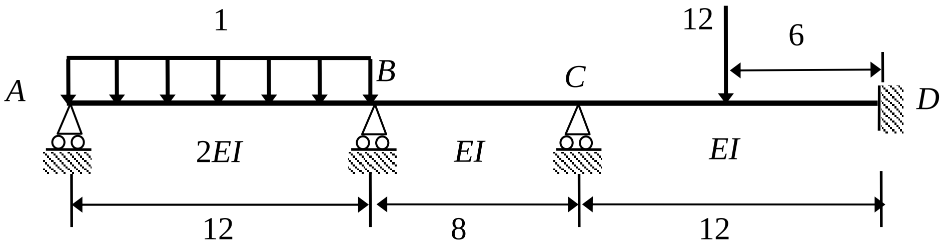
문제 1. 그림에서 보인 구조물을 응력법과 변위법에 의하여 해석하고 각 방법의 차이점에 대하여 기술하시오. (40 점)



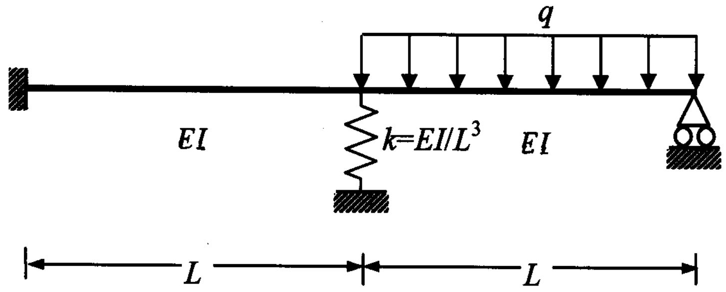
문제 2. 그림에서 보인 강절 뼈대 구조물에서 횡방향 변위가 발생하지 않는 하중의 위치 a 를 변위법에 의하여 구하시오. (30 점)



문제 3. 그림에서 보인 들보를 Gauss-Siedal Method 기초한 모멘트 분배법에 의하여 해석하고 B 점과 C 점에서의 회전각을 구하시오. 3 회 정도의 반복 계산을 수행하시오.



문제 1. 그림에서 보인 구조물을 응력법과 변위법에 의하여 해석하고 각 방법의 차이점에 대하여 기술하시오. (40 점)



응력법

2차 부정정 ; spring 지점과 roller 지점 제거

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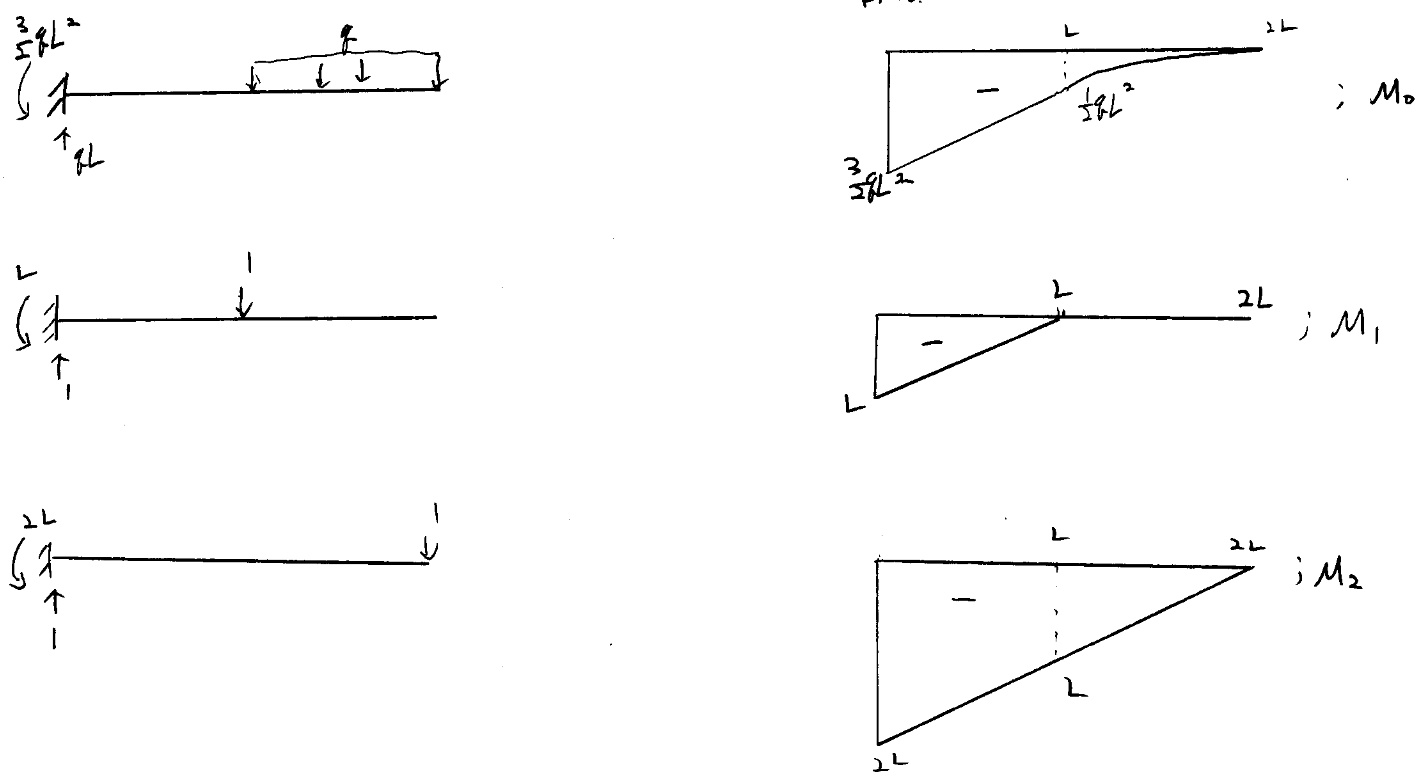


$$\begin{cases} \delta_{10} + \delta_{11} X_1 + \delta_{12} X_2 = -\dfrac{X_1}{k} \\ \delta_{20} + \delta_{21} X_1 + \delta_{22} X_2 = 0 \end{cases}$$

$$\delta_{10} = \frac{L}{6EI}\left\{(2)\left(-\frac{3}{2}qL^2\right) - \frac{1}{2}qL^2\right\}\cdot(-L) = \frac{7qL^4}{12EI}$$

$$\delta_{12} = \frac{L}{6EI}\cdot(-5L)(-L) = \frac{5L^3}{6EI}$$

$$\delta_{20} = \frac{qL^4}{EI}\cdot\frac{19}{12} + \frac{qL^4}{8EI} = \frac{41qL^4}{24EI}$$

$$\delta_{22} = \frac{2L}{3EI}\cdot(2L)\cdot(2L) = \frac{8L^3}{3EI}$$

$$\delta_{11} = \frac{L}{3EI}\cdot L\cdot L = \frac{L^3}{3EI}$$

//

$$\therefore \quad \frac{7}{12}\frac{qL^4}{EI} + \frac{L^3}{3EI}x_1 + \frac{5}{6}\frac{L^3}{EI}\cdot 2 = -\frac{L^3}{EI}x_1$$

$$\frac{41}{24}\frac{qL^4}{EI} + \frac{5L^3}{6EI}x_1 + \frac{8L^3}{3EI}x_2 = 0.$$

$$\Rightarrow \quad x_1 = R_B = -0.046qL \quad , \quad x_2 = R_c = -0.6208L.$$

변위법.

DOF ; θ_B , Δ_B (↓ ⓕ)

Fixed end force

$$M^f_{BC} = \frac{1}{8}qL^2 \qquad V^f_{BC} = -\frac{5}{8}qL.$$

$\theta_B \neq 0$

$$M'_{AB} = \frac{2EI}{L}\theta_B \ , \ M'_{BA} = \frac{4EI}{L}\theta_B \ , \ M'_{BC} = \frac{3EI}{L}\theta_B$$

$$V'_{BA} = \frac{6EI}{L^2}\theta_B \ , \ V'_{BC} = -\frac{3EI}{L^2}\theta_B$$

$\Delta_B \neq 0$

$$M^2_{AB} = M^2_{BA} = \frac{6EI}{L^2}\Delta_B \ , \ M^2_{BC} = -\frac{3EI}{L^2}\Delta_B$$

$$V^2_{BA} = \frac{12EI}{L^3}\Delta_B \ , \ V^2_{BC} = \frac{3EI}{L^3}\Delta_B \ , \ V_s = \frac{EI}{L^3}\Delta_B$$

$$\Sigma M_B = 0 \ ; \quad \frac{1}{8}qL^2 + \frac{7EI}{L}\theta_B + \frac{3EI}{L^2}\Delta_B = 0$$

$$\Sigma V_B = 0 \ ; \quad -\frac{5}{8}qL + \frac{3EI}{L^2}\theta_B + \frac{16EI}{L^3}\Delta_B = 0$$

$$\Rightarrow \quad \theta_B = -\frac{31qL^3}{824EI} \ , \ \Delta_B = \frac{38qL^4}{824EI}$$

대입하여 값 구하면.

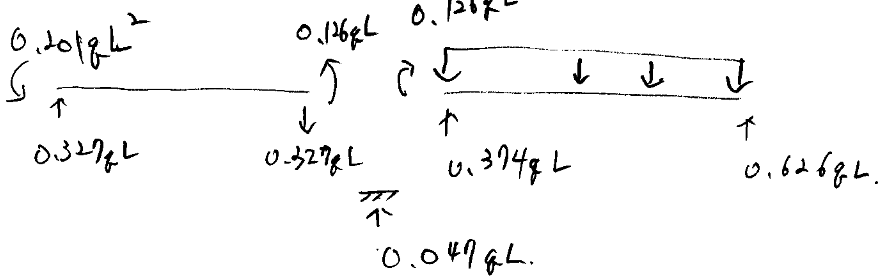
$M_{AB} = 0.201qL^2.$

$M_{BA} = 0.126qL^2.$

$M_{BC} = -0.126qL^2$

$V_s = 0.046qL$

free body diagram.



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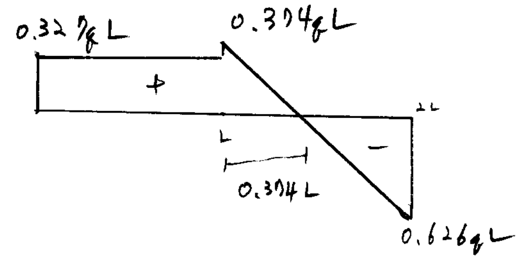

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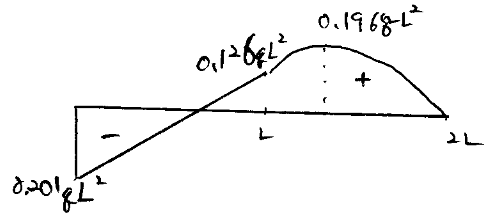
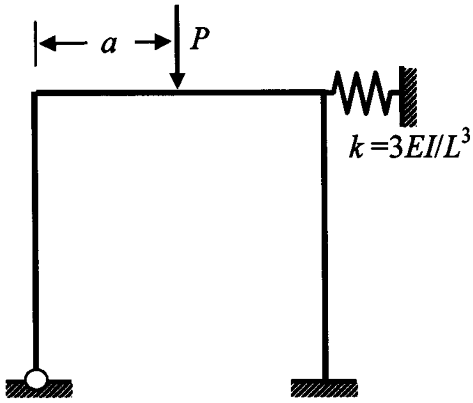


응력법 : Equilibrium 만족시키는 힘 중 Compatibility 만족하는 힘을 찾는것

변위법 : Compatibility 만족시키는 변위중 Equilibrium 만족하는 변위를 찾는것

문제 2. 그림에서 보인 강절 뼈대 구조물에서 횡방향 변위가 발생하지 않는 하중의 위치 a를 변위법에 의하여 구하시오. (30 점)



각 길이를 L로 가정. $b = L - a$.

DOF ; θ_B, θ_C, Δ

All Fixed ; $M^f_{BC} = \frac{Pab^2}{L^2}$, $M^f_{CB} = -\frac{Pa^2b}{L^2}$

$\theta_B \neq 0$

$M'_{BA} = \frac{3EI}{L}\theta_B$, $M'_{BC} = \frac{4EI}{L}\theta_B$, $M'_{CB} = \frac{2EI}{L}\theta_B$. $V'_{BA} = \frac{3EI}{L^2}\theta_B$

$\theta_C \neq 0$

$M^2_{BC} = \frac{2EI}{L}\theta_C$, $M^2_{CB} = \frac{4EI}{L}\theta_C$, $M^2_{CD} = \frac{4EI}{L}\theta_C$, $M^2_{DC} = \frac{2EI}{L}\theta_C$ $V^2_{CD} = \frac{6EI}{L^2}\theta_C$

$\Delta \neq 0$

$M^3_{BA} = \frac{3EI}{L^2}\Delta$, $V^3_{BA} = \frac{3EI}{L^3}\Delta$, $M^3_{CD} \cdot M^3_{DC} = \frac{6EI}{L^2}\Delta$, $V^3_{CD} = \frac{12EI}{L^3}\Delta$, $\underline{V_s} = k \cdot \Delta = \frac{3EI}{L^3}\Delta$

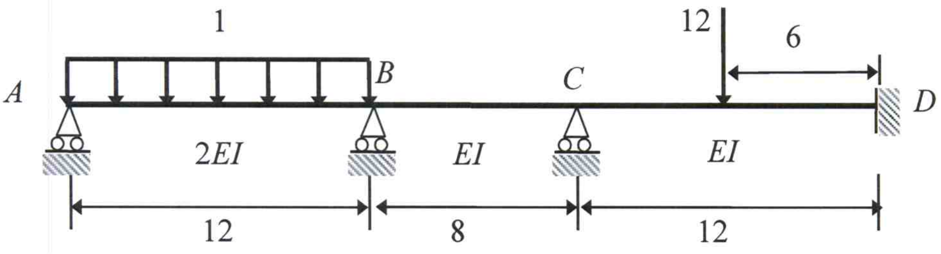
$\Sigma M_B = 0$; $\frac{Pab^2}{L^2} + \frac{7EI}{L}\theta_B + \frac{2EI}{L}\theta_C + \frac{3EI}{L^2}\Delta = 0$

$\Sigma M_C = 0$; $-\frac{Pa^2b}{L^2} + \frac{2EI}{L}\theta_B + \frac{8EI}{L}\theta_C + \frac{6EI}{L^2}\Delta = 0$ $\Rightarrow$ $\frac{Pab^2}{L} = \frac{12EI}{L}\theta_C$ $\Rightarrow$ $b = 3a$

$\Sigma V = 0$; $\frac{3EI}{L^2}\theta_B + \frac{6EI}{L^2}\theta_C + \frac{18EI}{L^3}\Delta = 0$ $\frac{Pa^2b}{L} = \frac{4EI}{L}\theta_C$

$\therefore$ $a = \frac{L}{4}$ 일때 횡방향 변위가 0.

문제 3. 그림에서 보인 들보를 Gauss-Siedal Method 기초한 모멘트 분배법에 의하여 해석하고 B 점과 C 점에서의 회전각을 구하시오. 3 회 정도의 반복 계산을 수행하시오.



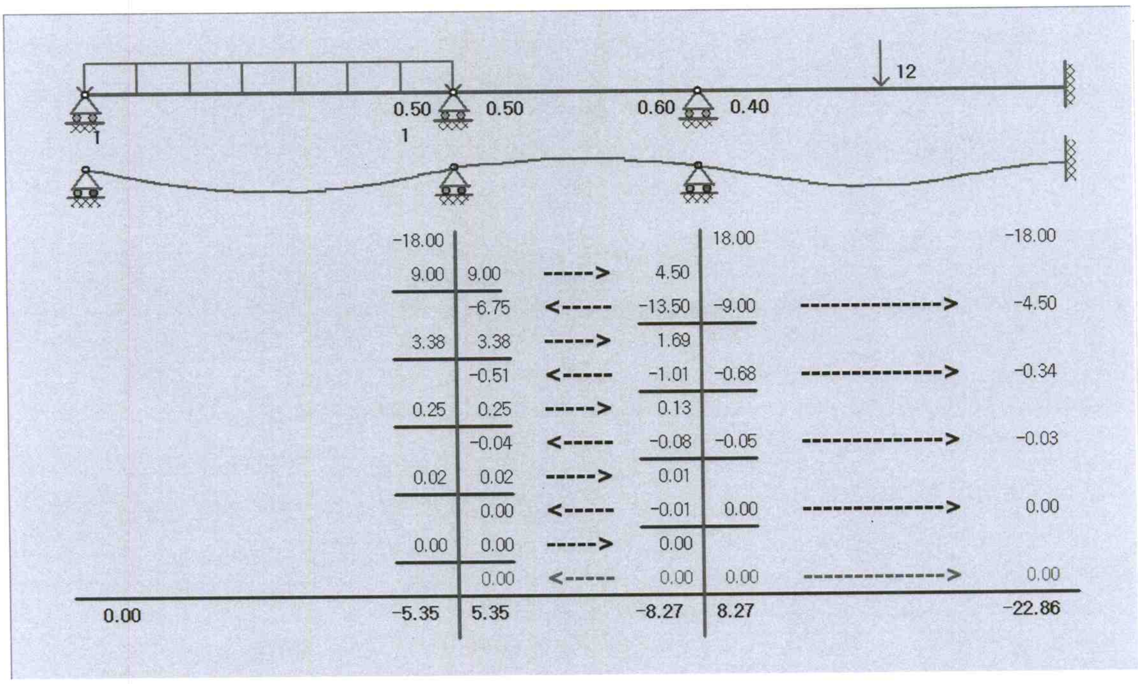
Fixed end moment

$$M^f_{CD} = -M^f_{DC} = \frac{PL}{8} = \frac{12\cdot 12}{8} = 18 \qquad M^f_{BA} = -\frac{1}{8}qL^2 = -\frac{1}{8}\cdot 12^2 = -18$$

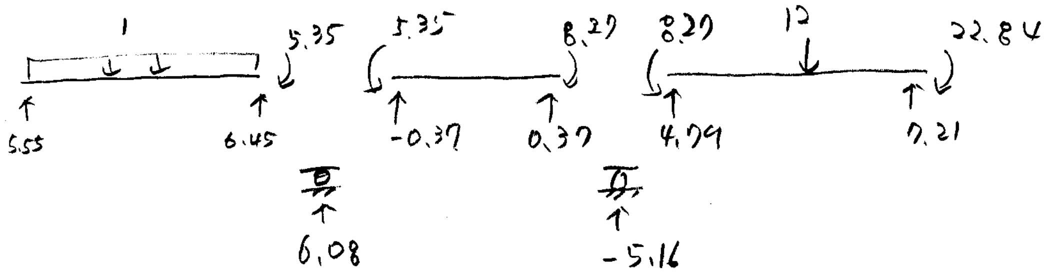
분배율

$$D_{BA} = \frac{\frac{6EI}{12}}{\frac{3\cdot 2EI}{12} + \frac{4EI}{8}} = \frac{\frac{6}{12}}{\frac{6}{12}+\frac{4}{8}} = 0.5 \qquad D_{BC} = 0.5$$

$$D_{CB} = \frac{\frac{4EI}{8}}{\frac{4EI}{8} + \frac{4EI}{12}} = \frac{\frac{4}{8}}{\frac{4}{8}+\frac{4}{12}} = 0.6\ , \quad D_{CD} = 0.4$$



freebody diagram



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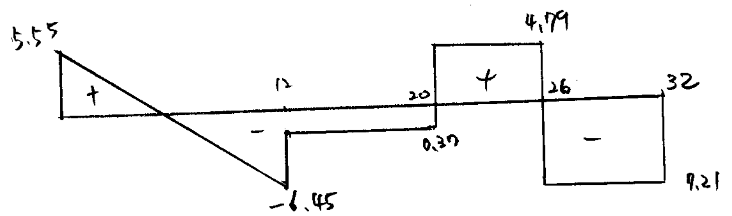

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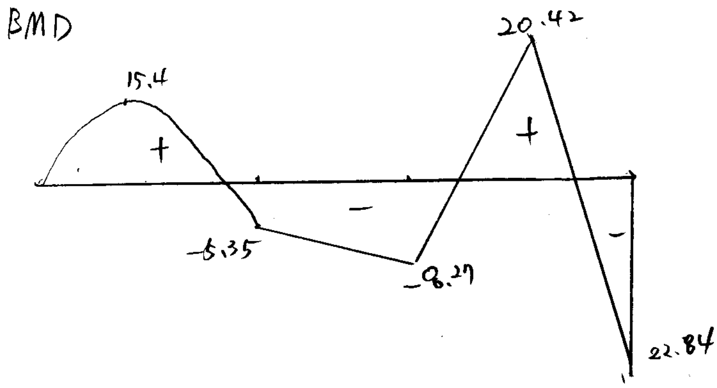


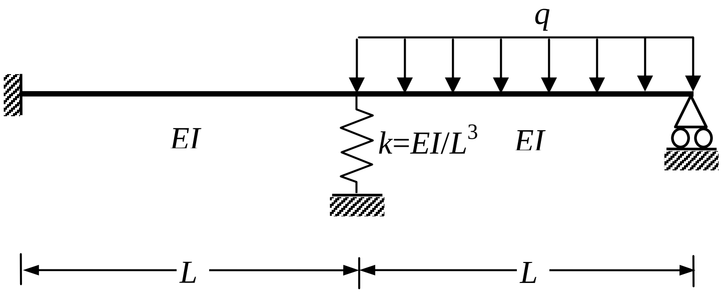
$$\Delta M_{BA} = \frac{6}{12}EI\,\theta_B = 9 + 3.395 + 0.254 + 0.019 \qquad \therefore \theta_B = \frac{25.396}{EI}$$

$$\Delta M_{CD} = \frac{4EI}{12}\theta_C = -9 - 0.675 - 0.051 \qquad \therefore \theta_C = -\frac{29.178}{EI}$$

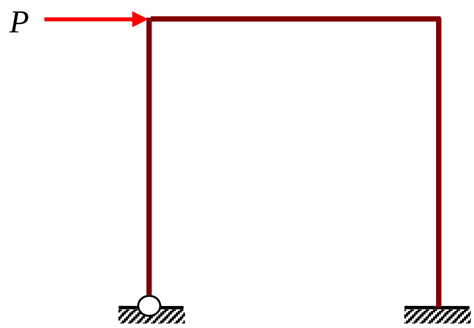
구조역학 II 중간고사 2

2006. 11. 13.

문제 1. 그림에서 보인 구조물을 최소일의 원리에 의하여 해석하고 각 지점의 반력, 모멘트도 및 전단력도를 구하여 변위법에 의한 정해와 비교하시오. 단 왼쪽 경간의 처짐은 2차 다항식으로, 오른쪽 경간의 처짐은 3차 다항식으로 가정하시오. (50점)

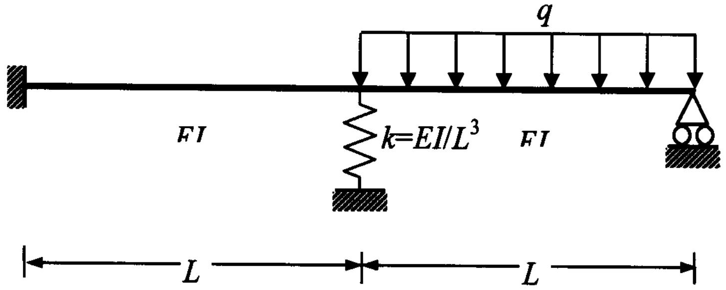


문제 2. 그림에서 보인 강절 뼈대 구조물을 모멘트 분배법으로 해석하시오. 단 모든 부재의 길이와 휨 강성은 동일하다. (30점)



문제 3. 어떤 들보가 평형상태에 있을 경우 최소일의 원리가 항상 성립함을 보이시오. (20점)

문제 1. 그림에서 보인 구조물을 최소일의 원리에 의하여 해석하고 각 지점의 반력, 모멘트도 및 전단력도를 구하여 변위법에 의한 정해와 비교하시오. 단 왼쪽 경간의 처짐은 2 차 다항식으로, 오른쪽 경간의 처짐은 3 차 다항식으로 가정하시오. (50 점)



$W_L = a_0 + a_1 x + a_2 x^2$

$W_R = b_0 + b_1 x + b_2 x^2 + b_3 x^3$

B.C.

$W_L(0) = 0 \;;\; a_0 = 0$, $\quad W_L'(0) = 0 \;;\; a_1 = 0$, $\quad W_L(L) = W_R(0) \;;\; L^2 a_2 = b_0$

$W_L'(L) = W_R'(0) \;;\; 2La_2 = b_1$, $\quad W_R(L) = 0 \;;\; L^2 a_2 + 2L^2 a_2 + L^2 b_2 + L^3 b_3 = 0$

$\rightarrow b_2 = -Lb_3 - 3a_2$

$\Rightarrow W_L = a_2 x^2 \quad , \quad W_R = L^2 a_2 + 2La_2 x - (Lb_3 + 3a_2)x^2 + b_3 x^3$

$W_L'' = 2a_2 \quad , \quad W_R'' = -6a_2 + b_3(6x - 2L)$

$-3a_2 + b_3(3x - L)$

$$\Pi_{total} = \frac{1}{2}\int_0^L (W_L'')^2 EI\,dx + \frac{1}{2}\int_0^L (W_R'')^2 EI\,dx + \frac{1}{2}\frac{EI}{L^3}(L^2 a_2)^2 - \int_0^L q W_R\,dx$$

$$= 2EI\int_0^L a_2^2\,dx + 2EI\int_0^L \left(9a_2^2 - 6a_2 b_3(3x - L) + b_3^2(9x^2 - 6Lx + L^2)\right) dx$$

$$+ \frac{EIL}{2}a_2^2 + \frac{qL^4}{12}b_3 - qL^3 a_2$$

$$= 2EILa_2^2 + 2EIL^3 b_3^2 - 6EIL^2 a_2 b_3 + 18EILa_2^2 + \frac{EIL}{2}a_2^2 + \frac{qL^4}{12}b_3 - qL^3 a_2$$

$$\frac{\partial \pi}{\partial a_2} = EI\left(-6L^2 b_3 + 4 L a_2\right) - qL^3 = 0$$

$$\frac{\partial \pi}{\partial b_3} = EI\left(4L^3 b_3 - 6L^2 a_2\right) + \frac{qL^4}{12} = 0$$

$$a_2 = 0.0273\frac{qL^2}{EI} \qquad b_3 = 0.0202\frac{qL}{EI}$$

$$w_L = 0.0273\frac{qL^2}{EI}x^2$$

$$w_R = 0.0273\frac{qL^4}{EI} + 0.0546\frac{qL^3}{EI}x - 0.1021\frac{qL^2}{EI}x^2 + 0.0202\frac{qL}{EI}x^3$$

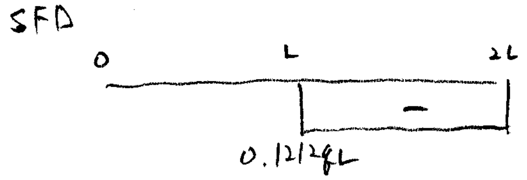
$$M_L = -EIw_L'' = -0.0546qL^2$$

$$M_R = -EIw_R'' = 0.2042qL^2 - 0.1212qLx$$

$$V_L = -EIw_L''' = 0$$

$$V_R = -EIw_R''' = -0.1212qL$$

SFD



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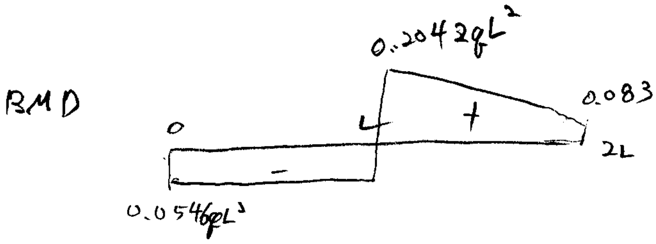


변위법

DOF ; θ_B , Δ_B ($\downarrow\oplus$)

Fixed end force

$$M_{BC}^f = \frac{1}{8}qL^2 \qquad V_{BC}^f = -\frac{5}{8}qL$$

$\theta_B \neq 0$

$$M_{AB}^1 = \frac{2EI}{L}\theta_B, \quad M_{BA}^1 = \frac{4EI}{L}\theta_B, \quad M_{BC}^1 = \frac{3EI}{L}\theta_B, \quad V_{BA}^1 = \frac{6EI}{L^2}\theta_B, \quad V_{BC}^1 = -\frac{3EI}{L^2}\theta_B$$

$\Delta_B \neq 0$

$$M_{AB}^2 = M_{BA}^2 = \frac{6EI}{L^2}\Delta_B \quad M_{BC}^2 = -\frac{3EI}{L^2}\Delta_B \quad V_{BA}^2 = \frac{12EI}{L^3}\Delta_B, \quad V_{BC}^2 = \frac{3EI}{L^3}\Delta_B, \quad \underline{V_S} = \frac{EI}{L^3}\Delta_B$$

$$\Sigma M_B = 0 \; ; \quad \frac{1}{8}qL^2 + \frac{9EI}{L}\theta_B + \frac{3EI}{L^2}\Delta_B = 0$$

$$\Sigma V_B = 0 \; ; \quad -\frac{5}{8}qL + \frac{3EI}{L^2}\theta_B + \frac{16EI}{L^3}\Delta_B = 0$$

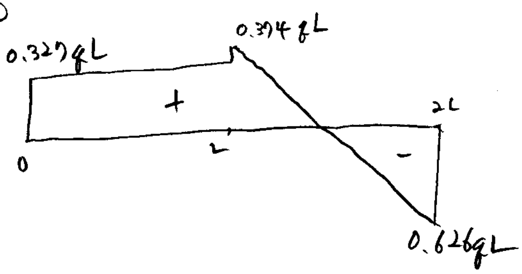
$$\Rightarrow \theta_B = -\frac{31qL^3}{824EI} \; , \quad \Delta_B = \frac{38qL^4}{824EI}$$

대입하면

$$M_{AB} = 0.2018L^2 \qquad M_{BA} = 0.126qL^2$$

$$M_{BC} = -0.126qL^2 \qquad V_s = 0.046qL$$

SFD



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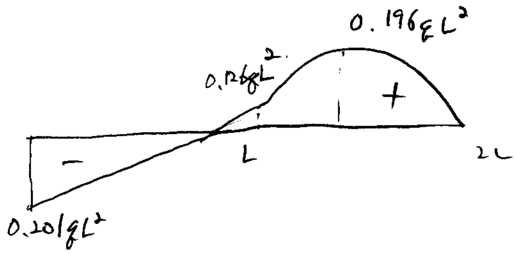
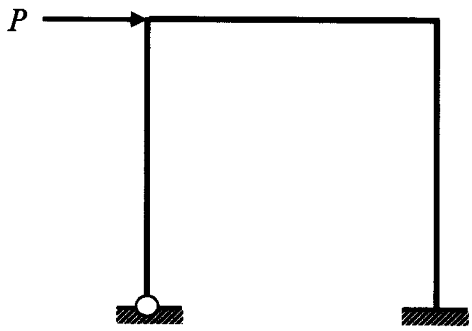


문제 2. 그림에서 보인 강절 뼈대 구조물을 모멘트 분배법으로 해석하시오. 단 모든 부재의 길이와 휨 강성은 동일하다. (30 점)



길이 ; L

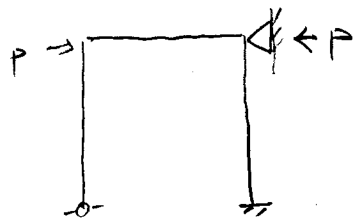
강성 ; EI.

DOF ; θ_B, θ_C, Δ ($\rightarrow \oplus$)

All fixed ; none.

$D_{BA} = 0.43$. $D_{BC} = 0.57$ $D_{CB} = 0.5$ $D_{CD} = 0.5$

i) no sidesway



ii) arbitrary sidesway

0.43	0.57		0.5	0.5		
5				10		10
-2.15	-2.85	→	-1.43			
	-2.15	←	-4.29	-4.29	→	-2.15
0.92	1.23	→	0.62			
	-0.16	←	-0.31	-0.31	→	-0.16
0.07	0.09	→	0.05			
	-0.02	←	-0.03	-0.03	→	-0.02
0.01	0.01					
3.85	-3.85		-5.39	5.37		7.67

Total moment = First moment $\alpha \times$ Second moment

$$= -P + \alpha \cdot \frac{16.89}{L} = 0$$

$$\therefore \alpha = \frac{PL}{16.89}$$

$\therefore$ Total moment

$M_{AB} = 0 \quad M_{BA} = 0 + \frac{PL}{16.89} \cdot (3.85) = 0.23PL$

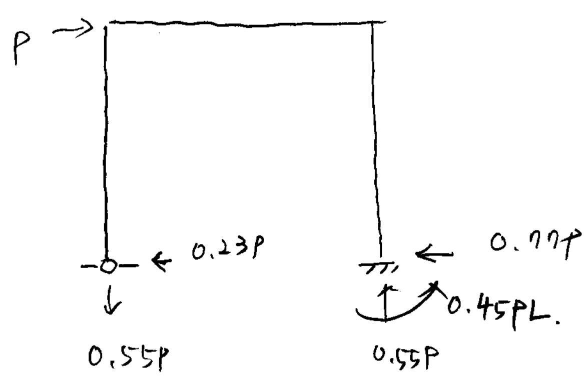
$M_{BC} = 0 + \frac{PL}{16.89} \cdot (-3.85) = -0.23pL$

$M_{CB} = 0 + \frac{PL}{16.89} \cdot (-5.39) = -0.32PL$

$M_{CD} = 0 + \frac{PL}{16.89} \cdot (5.37) = 0.32PL$

$M_{DC} = 0 + \frac{PL}{16.89} \cdot (7.67) = 0.45PL.$

반력;



문제 3. 어떤 들보가 평형상태에 있을 경우 최소일의 원리가 항상 성립함을 보이시오. (20점)

평형상태에 있을 경우

$$EI\frac{d^2M^e}{dx^2} = -q \quad \text{or} \quad EI\frac{d^4w^e}{dx^4} = q$$

가 성립한다.

$w = w^e + \overline{w}$ 라고 할 때

$$\Pi^h = \frac{1}{2}\int_0^l (\frac{d^2(w^e+\overline{w})}{dx^2} EI \frac{d^2(w^e+\overline{w})}{dx^2})dx - \int_0^l (w^e+\overline{w})qdx$$

$$= \frac{1}{2}\int_0^l (\frac{d^2w^e}{dx^2} EI \frac{d^2w^e}{dx^2})dx - \int_0^l w^e qdx + \frac{1}{2}\int_0^l (\frac{d^2\overline{w}}{dx^2} EI \frac{d^2\overline{w}}{dx^2})dx + \int_0^l (\frac{d^2\overline{w}}{dx^2} EI \frac{d^2w^e}{dx^2})dx - \int_0^l \overline{w}qdx$$

$$= \Pi^e + \frac{1}{2}\int_0^l (\frac{d^2\overline{w}}{dx^2} EI \frac{d^2\overline{w}}{dx^2})dx + \int_0^l (-EI\frac{d^2\overline{w}}{dx^2})\frac{1}{EI}(-EI\frac{d^2w^e}{dx^2})dx - \int_0^l \overline{w}qdx$$

$$= \Pi^e + \frac{1}{2}\int_0^l (\frac{d^2\overline{w}}{dx^2} EI \frac{d^2\overline{w}}{dx^2})dx + \int_0^l \frac{\overline{M}M^e}{EI}dx - \int_0^l \overline{w}qdx$$

$$= \Pi^e + \frac{1}{2}\int_0^l (\frac{d^2\overline{w}}{dx^2} EI \frac{d^2\overline{w}}{dx^2})dx + \int_0^l \frac{d^2\overline{w}}{dx^2} EI \frac{d^2w^e}{dx^2}dx - \int_0^l \overline{w}qdx$$

$$= \Pi^e + \frac{1}{2}\int_0^l (\frac{d^2\overline{w}}{dx^2} EI \frac{d^2\overline{w}}{dx^2})dx + \int_0^l \overline{w}EI\frac{d^4w^e}{dx^4}dx$$

$$- \int_0^l \overline{w}qdx - \frac{d\overline{w}}{dx}EI\frac{d^2w^e}{dx^2}\Big|_0^l + \overline{w}EI\frac{d^3w^e}{dx^3}\Big|_0^l \quad (BoundaryCondition)$$

$$= \Pi^e + \frac{1}{2}\int_0^l (\frac{d^2\overline{w}}{dx^2} EI \frac{d^2\overline{w}}{dx^2})dx + \int_0^l \overline{w}EI\frac{d^4w^e}{dx^4}dx - \int_0^l \overline{w}qdx \quad (Equilibrium\ Equation)$$

$$= \Pi^e + \frac{1}{2}\int_0^l (\frac{d^2\overline{w}}{dx^2} EI \frac{d^2\overline{w}}{dx^2})dx \ge \Pi^e \quad \text{for all virtual } \overline{w}$$

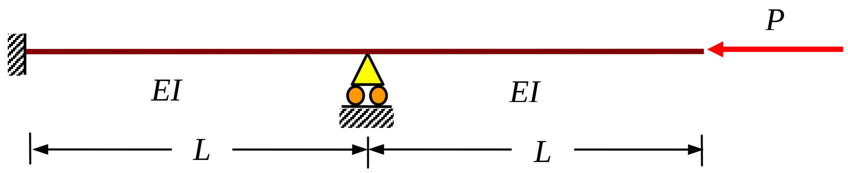
∴ 들보가 평형상태에 있을 경우 최소일의 원리는 항상 성립한다.

School of Civil, Urban and Geosystem Engineering
Seoul National University
Kwanak-Gu, Seoul 151-742, Korea

구조 역학 II 기말고사

2006. 12. 18

문제 1. 그림과 같은 들보 구조물의 좌굴 하중을 에너지법에 의하여 구하시오. (40 점)

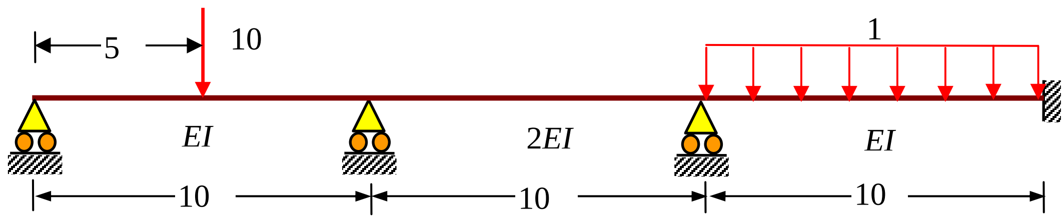


a) 각 경간에서 주어진 변위 경계 조건을 만족하도록 처짐곡선을 왼쪽 경간에서는 3 차 다항식으로, 오른쪽 경간에서는 2 차 다항식으로 가정하시오. (10 점)

b) 중앙 지점에서의 회전각에 대한 적합 조건을 다항식의 계수로 표시하시오. (10 점)

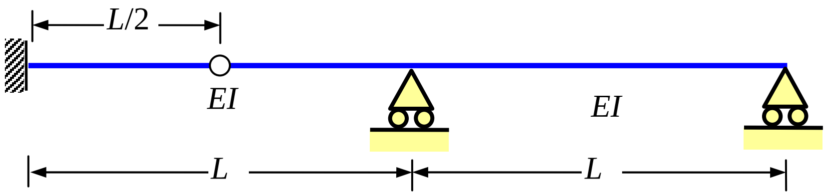
c) 에너지법을 이용하여 좌굴 하중을 구하시오. (10 점+10 점)

문제 2. 그림에서 보인 구조물을 Gauss-Siedal 방법에 기초한 모멘트 분배법에 의하여 해석하여, 지점 반력을 구하시오. 또한 왼쪽에서 두번째 지점에서의 회전각을 구하시오. 3 회의 반복 계산을 수행하시오. (30)

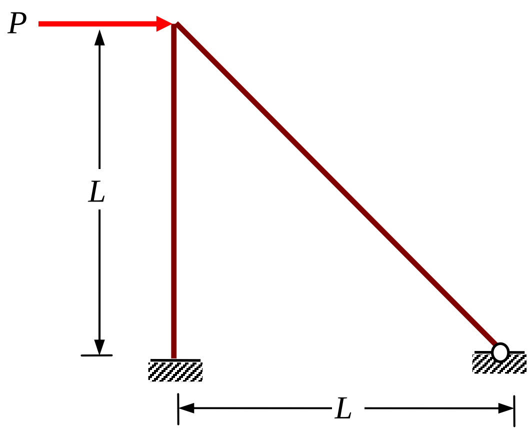


문제 3. 문제 2 에서 주어진 구조물을 요각법으로 해석하여 각 지점의 회전각을 구하시오. (30 점)

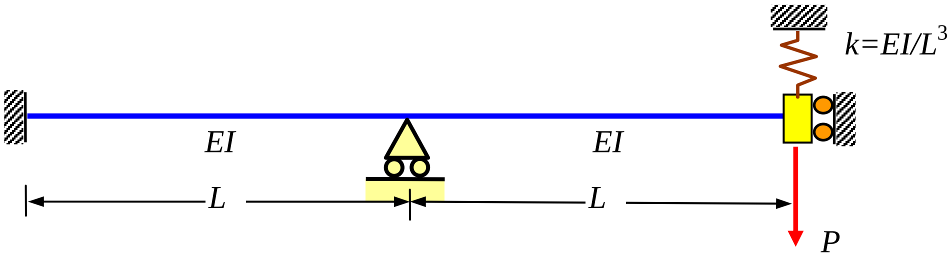
문제 4. 다음과 같은 Gerber 들보의 왼쪽 고정단 모멘트에 대한 영향선을 구하시오. (30)



문제 5. 오른쪽 그림에서 보인 프레임 구조물을 요각법에 의하여 해석하시오. 수직재의 축방향 변형은 무시하고 사재의 축방향 변형은 고려하시오. 단 유도된 강성도 방정식을 풀필요는 없다. 모든 부재의 휨강성은 *EI* 이고 사재의 축방향 강성은 *EA* 이다. (40 점)



문제 6. 그림과 같은 구조물을 직접 강성도법을 적용한 매트릭스 구조해석법에 의하여 해석하여 각 자유도에서의 처짐을 구하고 각 지점에서 발생하는 반력을 구하시오. 보부재의 강성도 행렬은 다음과 같다. (30)



$$\begin{pmatrix} V_e^L \\ M_e^L \\ V_e^R \\ M_e^R \end{pmatrix} = \frac{EI_e}{L_e}\begin{bmatrix} \frac{12}{L_e^2} & \frac{6}{L_e} & -\frac{12}{L_e^2} & \frac{6}{L_e} \\ \frac{6}{L_e} & 4 & -\frac{6}{L_e} & 2 \\ -\frac{12}{L_e^2} & -\frac{6}{L_e} & \frac{12}{L_e^2} & -\frac{6}{L_e} \\ \frac{6}{L_e} & 2 & -\frac{6}{L_e} & 4 \end{bmatrix}\begin{pmatrix} w_e^L \\ \theta_e^L \\ w_e^R \\ \theta_e^R \end{pmatrix}$$

수고하셨습니다. 즐거운 겨울 방학 보내세요 !!!

〈문제 1〉

(a) $w_L = a_1 x^3 + b_1 x^2 + c_1 x + d_1$, $w_R = a_2 x^2 + b_2 x + c_2$

- <u>Displacement Boundary Condition</u>

$w_L(0) = 0 \rightarrow \underline{d_1 = 0}$ $\qquad w_L(L) = 0 \rightarrow a_1 L^3 + b_1 L^2 = 0 \quad \underline{b_1 = -a_1 L}$

$w_L'(0) = 0 \rightarrow \underline{c_1 = 0}$ $\qquad w_R(0) = 0 \rightarrow \underline{c_2 = 0}$

$$\therefore \begin{cases} w_L = a_1 x^3 - a_1 L x^2 \\ w_R = a_2 x^2 + b_2 x \end{cases}$$

(b) $\underline{w_L'(L) = w_R'(0)} \rightarrow 3a_1 L^2 - 2a_1 L^2 = \underline{b_2 = a_1 L^2}$

$$\therefore \begin{cases} w_L = a_1 x^3 - a_1 L x^2 \\ w_R = a_2 x^2 + a_1 L^2 x \end{cases}$$

$$\begin{cases} w_L' = 3a_1 x^2 - 2a_1 L x \\ w_R' = 2a_2 x + a_1 L^2 \end{cases}$$

$$\begin{cases} w_L'' = 6a_1 x - 2a_1 L \\ w_R'' = 2a_2 \end{cases}$$

(c) <u>Total Potential Energy</u>

$$\Pi_{total} = \frac{1}{2}\int_0^{2L} \frac{d^2(w^e+\bar{w})}{dx^2} EI \frac{d^2(w^e+\bar{w})}{dx^2} dx \quad \frac{1}{2}\int_0^{2L} \frac{d(w^e+\bar{w})}{dx} p \frac{d(w^e+\bar{w})}{dx} dx$$

$$- \int_0^{2L} (w^e+\bar{w}) q \, dx$$

$$= \underbrace{\frac{1}{2}\int_0^{2L} \frac{d^2 w^e}{dx^2} EI \frac{d^2 w^e}{dx^2} dx - \frac{1}{2}\int_0^{2L} \frac{dw^e}{dx} p \frac{dw^e}{dx} dx - \int_0^{2L} w^e q \, dx}_{(=\Pi^{exact})}$$

$$+ \frac{1}{2}\int_0^{2L} \frac{d^2\bar{w}}{dx^2} EI \frac{d^2\bar{w}}{dx^2} dx - \frac{1}{2}\int_0^{2L} \frac{d\bar{w}}{dx} p \frac{d\bar{w}}{dx} dx - \int_0^{2L} \bar{w} q \, dx$$

$$+ \int_0^{2L} \frac{d^2 w^e}{dx^2} EI \frac{d^2 \bar{w}}{dx^2} dx - \int_0^{2L} \frac{dw^e}{dx} p \frac{d\bar{w}}{dx} dx$$

$$\left(\begin{aligned} \int u \cdot v'''' &= u v''' - \int u' v''' = u v''' - u' v'' + \int u'' v'' \\ &= u v''' - u' v'' + u'' v' - \int u''' v' \\ &= u v''' - u' v'' + u'' v' - u''' v + \int u'''' v \end{aligned} \right)$$

$$= \Pi^{exact} + \frac{1}{2}\int_0^{2L} \frac{d^2\bar{w}}{dx^2} EI \frac{d^2\bar{w}}{dx^2}\,dx - \frac{1}{2}\int_0^{2L} \frac{d\bar{w}}{dx} P \frac{d\bar{w}}{dx}\,dx$$

$$- \int_0^{2L} \bar{w} q\,dx + \cancel{\frac{d^2w^e}{dx^2} EI \frac{d\bar{w}}{dx}\Big|_0^{2L}}^{\,0} - \cancel{\frac{d^3w^e}{dx^3} EI \bar{w}\Big|_0^{2L}}^{\,0} + \int \frac{d^4w^e}{dx^4} EI \bar{w}\,dx$$

$$- \cancel{\frac{dw^e}{dx} P \bar{w}\Big|_0^{2L}}^{\,0} + \int_0^{2L} \frac{d^2w^e}{dx^2} P \bar{w}\,dx$$

$$= \Pi^{exact} + \frac{1}{2}\int_0^{2L} \frac{d^2\bar{w}}{dx^2} EI \frac{d^2\bar{w}}{dx^2}\,dx - \frac{1}{2}\int_0^{2L} \frac{d\bar{w}}{dx} P \frac{d\bar{w}}{dx}$$

$$+ \int_0^{2L} \bar{w}\left(EI\frac{d^4w^e}{dx^4} + P\frac{d^2w^e}{dx^2} - q\right)dx$$

$$= \Pi^{exact} + \underline{\frac{1}{2}\int_0^{2L} \frac{d^2\bar{w}}{dx^2} EI \frac{d^2\bar{w}}{dx^2}\,dx - \frac{1}{2}\int_0^{2L} \frac{d\bar{w}}{dx} P \frac{d\bar{w}}{dx}\,dx} \quad (= \Pi^{RR})$$

- The principle of minimum potential energy holds if and only if

$$\int_0^{2L} \left(\frac{d^2\bar{w}}{dx^2} EI \frac{d^2\bar{w}}{dx^2} - \frac{d\bar{w}}{dx} P \frac{d\bar{w}}{dx}\right) dx > 0 \qquad \text{for all admissible } \bar{w}$$

- The principle of minimum potential energy is not valid for the following cases

$$\int_0^{2L} \left(\frac{d^2\bar{w}}{dx^2} EI \frac{d^2\bar{w}}{dx^2} - \frac{d\bar{w}}{dx} P \frac{d\bar{w}}{dx}\right) dx \leq 0 \qquad \text{for some } \bar{w}$$

- The critical status of a structure is defined as

$$\int_0^{2L} \left(\frac{d^2\bar{w}}{dx^2} EI \frac{d^2\bar{w}}{dx^2} - \frac{d\bar{w}}{dx} P \frac{d\bar{w}}{dx}\right) dx = 0$$

- Approximation using the principle of minimum potential energy
  - Approximation of displacement : $\bar{w} = \sum_{i=1}^{n} a_i g_i$
  - Critical Status

$$\int_0^{2L} \left(\sum_{i=1}^{n} a_i g_i''\right) EI \left(\sum_{j=1}^{n} a_j g_j''\right) dx - \int_0^{2L} \left(\sum_{i=1}^{n} a_i g_i'\right) P \left(\sum_{j=1}^{n} a_j g_j'\right) dx$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} a_i \int_0^{2L} g_i'' EI g_j''\,dx\, a_j - P\sum_{i=1}^{n}\sum_{j=1}^{n} a_i \int_0^{2L} g_i' g_j'\,dx\, a_j$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} a_i K_{ij} a_j - P\sum_{i=1}^{n}\sum_{j=1}^{n} a_i K_{ij}^G a_j = (\mathbf{a})^T(\mathbf{K} - P\mathbf{K}^G)(\mathbf{a})^T = 0$$

$$\rightarrow \text{Det}(\mathbf{K}-P\mathbf{K}_G)=0$$

$$w_L = a_1(-Lx^2+x^3) \quad \rightarrow \quad g_L = -Lx^2+x^3, \quad g_L' = -2Lx+3x^2, \quad g_L'' = -2L+6x$$

$$w_R = a_1 L^2 x + a_2 x^2 \quad \rightarrow \quad \begin{cases} g_{R1} = L^2 x, & g_{R1}' = L^2, & g_{R1}'' = 0 \\ g_{R2} = x^2, & g_{R2}' = 2x, & g_{R2}'' = 2 \end{cases}$$

$$K_L = \int_0^L (g_L'')^2\,dx = \int_0^L (-2L+6x)^2\,dx = \int_0^L (4L^2-24Lx+36x^2)\,dx$$

$$= \left(4L^2x - 12Lx^2 + 12x^3\right)\Big|_0^L = 4L^3$$

$$K_L^G = \int_0^L (g_L')^2\,dx = \int_0^L (-2Lx+3x^2)^2\,dx = \int_0^L (4L^2x^2-12Lx^3+9x^4)\,dx$$

$$= \left(\frac{4}{3}L^2x^3 - 3Lx^4 + \frac{9}{5}x^5\right)\Big|_0^L = \frac{2}{15}L^5$$

$$(K_R)_{11} = \int_0^L (g_{R1}'')^2\,dx = \int_0^L 0\,dx = 0$$

$$(K_R)_{12} = (K_R)_{21} = \int_0^L (g_{R1}'')(g_{R2}'')\,dx = \int_0^L 0\,dx = 0$$

$$(K_R)_{22} = \int_0^L (g_{R2}'')^2\,dx = \int_0^L 4\,dx = 4L$$

$$(K_R)_{11}^G = \int_0^L (g_{R1}')^2\,dx = \int_0^L L^4\,dx = L^5$$

$$(K_R)_{12}^G = (K_R)_{21}^G = \int_0^L (g_{R1}')(g_{R2}')\,dx = \int_0^L 2L^2x\,dx = \left(L^2x^2\right)\Big|_0^L = L^4$$

$$(K_R)_{22}^G = \int_0^L (g_{R2}')^2\,dx = \int_0^L 4x^2\,dx = \left(\frac{4}{3}x^3\right)\Big|_0^L = \frac{4}{3}L^3$$

$$\det\left(EI\begin{bmatrix} K_L+(K_R)_{11} & (K_R)_{12} \\ (K_R)_{21} & (K_R)_{22} \end{bmatrix} - P\begin{bmatrix} K_L^G+(K_R)_{11}^G & (K_R)_{12}^G \\ (K_R)_{21}^G & (K_R)_{22}^G \end{bmatrix}\right) = 0$$

$$\det\left(EI\begin{bmatrix} 4L^3 & 0 \\ 0 & 4L \end{bmatrix} - P\begin{bmatrix} \frac{2}{15}L^5+L^5 & L^4 \\ L^4 & \frac{4}{3}L^3 \end{bmatrix}\right) = 0$$

$$\mathrm{Det}\left(EI\begin{bmatrix} 4L^3 & 0 \\ 0 & 4L \end{bmatrix} - \frac{P}{30}\begin{bmatrix} 34L^5 & 30L^4 \\ 30L^4 & 40L^3 \end{bmatrix}\right) = 0$$

$$\begin{vmatrix} 4L^3 - \alpha(34L^5) & -\alpha(30L^4) \\ -\alpha(30L^4) & 4L-\alpha(40L^3) \end{vmatrix} = 0 \quad , \quad \alpha = \frac{P}{30EI}$$

$$\{4L^3 - \alpha(34L^5)\}\{4L - \alpha(40L^3)\} - \{\alpha(30L^4)\}^2 = 0$$

$$(1360 - 900)L^8\alpha^2 - 296L^6\alpha + 16L^4 = 0$$

$$L^4(460L^4\alpha^2 - 296L^2\alpha + 16) = 0$$

$$L^4(460(L^2\alpha)^2 - 296(L^2\alpha) + 16) = 0$$

$$L^2\alpha = 0.583909798 \quad \text{or} \quad 0.059568462$$

$$\alpha = \frac{P}{30EI} = \frac{0.5839}{L^2} \quad \text{or} \quad \frac{0.05957}{L^2}$$

$$P = 17.52\frac{EI}{L^2} \quad \text{or} \quad 1.787\frac{EI}{L^2}$$

〈문제 2〉

- DOF : θ_B . θ_C

- All fixed

$$M^f_{BA} = \left(-\frac{3}{16}\right)\times 10\times 10 = -18.75$$

$$M^f_{CB} = -M^f_{BC} = \frac{1}{12}\times 10\times 10 = 8.33$$

- 분배율

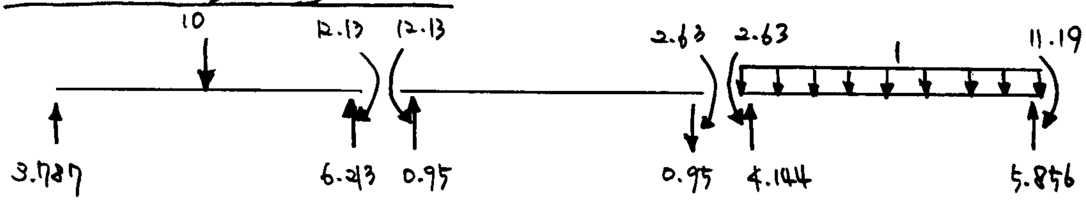
$$D_{BA} = \frac{3EI/10}{3EI/10 + 8EI/10} = 0.27, \quad D_{BC} = 1 - D_{BA} = 0.73$$

$$D_{CB} = \frac{8EI/10}{8EI/10 + 4EI/10} = 0.67, \quad D_{CD} = 1 - D_{CB} = 0.33$$

- MDM (Moment Distribution Method)

BA	BC	CB	CD	DC
0.27	0.73	0.67	0.33	
-18.75			8.33	→ -8.33
5.06	13.69 →	6.85		
	-5.09 ←	-10.17	-5.01 →	-2.51
1.37	3.72 →	1.86		
	-0.63 ←	-1.25	-0.61 →	-0.31
0.17	0.46 →	0.23		
	-0.08 ←	-0.15	-0.08 →	-0.04
0.02	0.06			
-12.13	12.13	-2.63	2.63	-11.19

- FBD (Freebody Diagram)



- Reactions

$R_A = 3.787$ (↑) , $R_B = 7.163$ (↑) , $R_C = 3.194$ (↑), $R_D = 5.856$ (↑)

$M_D = 11.19$ (↺)

- 회전각

$$\frac{3EI}{10}\theta_B = 5.06 + 1.37 + 0.17 + 0.02 = 6.62 \rightarrow \theta_B = \frac{10\times 6.62}{3EI} = \frac{22.07}{EI}$$

< 문제 3 >

Solution 1

- DOF : θ_B . θ_C

- Analysis

1) All fixed

$$M_{BA}^f = \left(-\frac{3}{16}\right) \times 10 \times 10 = -18.75 \qquad M_{CB}^f = -M_{BC}^f = \frac{1}{12} \times 10 \times 10 = 8.33$$

2) $\theta_B \neq 0$.

$$M_{BA}^1 = \frac{3EI}{10}\theta_B \, . \quad M_{BC}^1 = \frac{8EI}{10}\theta_B \, . \quad M_{CB}^1 = \frac{4EI}{10}\theta_B$$

3) $\theta_C \neq 0$.

$$M_{BC}^2 = \frac{4EI}{10}\theta_C \, . \quad M_{CB}^2 = \frac{8EI}{10}\theta_C \, . \quad M_{CD}^2 = \frac{4EI}{10}\theta_C \, . \quad M_{DC}^2 = \frac{2EI}{10}\theta_C$$

- Stiffness Equation

$$\Sigma M_B^i = 0 \; ; \; M_{BA}^f + M_{BA}^1 + M_{BC}^1 + M_{BC}^2 = 0$$

$$-18.75 + \left(\frac{3EI}{10} + \frac{8EI}{10}\right)\theta_B + \frac{4EI}{10}\theta_C = 0$$

$$\Sigma M_C^i = 0 \; ; \; M_{CB}^f + M_{CB}^1 + M_{CB}^2 + M_{CD}^2 = 0$$

$$8.33 + \frac{4EI}{10}\theta_B + \left(\frac{8EI}{10} + \frac{4EI}{10}\right)\theta_C = 0$$

$$\begin{cases} \frac{11EI}{10}\theta_B + \frac{4EI}{10}\theta_C = 18.75 \\ \frac{4EI}{10}\theta_B + \frac{12EI}{10}\theta_C = -8.33 \end{cases}$$

$$\theta_B = \frac{22.27}{EI} \, , \quad \theta_C = -\frac{14.36}{EI}$$

$$M_A = \frac{2EI}{10}(2\theta_A + \theta_B) + 12.5 = 0$$

$$\theta_A = -\frac{42.38}{EI}$$

$\theta_D = 0$ (∵ 지점 D는 고정단 이므로)

solution 2

- DOF : $\theta_A . \theta_B . \theta_c$

- Analysis

1) All fixed

$$M_{AB}^f = -M_{BA}^f = \frac{1}{8} \times 10 \times 10 = 12.5$$

$$M_{CD}^f = -M_{DC}^f = \frac{1}{12} \times 10 \times 10 = 8.33$$

2) $\theta_A \neq 0$

$$M_{AB}^1 = \frac{4EI}{10}\theta_A \qquad M_{BA}^1 = \frac{2EI}{10}\theta_A$$

3) $\theta_B = 0$

$$M_{AB}^2 = \frac{2EI}{10}\theta_B . \qquad M_{BA}^2 = \frac{4EI}{10}\theta_B . \qquad M_{BC}^2 = \frac{8EI}{10}\theta_B . \qquad M_{CB}^2 = \frac{4EI}{10}\theta_B$$

4) $\theta_c = 0$

$$M_{BC}^3 = \frac{4EI}{10}\theta_c . \qquad M_{CB}^3 = \frac{8EI}{10}\theta_c . \qquad M_{CD}^3 = \frac{4EI}{10}\theta_c . \qquad M_{DC}^3 = \frac{2EI}{10}\theta_c$$

- Stiffness Equation

$$\Sigma M_A^i = 0 ; \quad M_{AB}^f + M_{AB}^1 + M_{AB}^2 = 0$$

$$12.5 + \frac{4EI}{10}\theta_A + \frac{2EI}{10}\theta_B = 0$$

$$\Sigma M_B^i = 0 ; \quad M_{BA}^f + M_{BA}^1 + M_{BA}^2 + M_{BC}^2 + M_{BC}^3 = 0$$

$$-12.5 + \frac{2EI}{10}\theta_A + \left(\frac{4EI}{10} + \frac{8EI}{10}\right)\theta_B + \frac{4EI}{10}\theta_c = 0$$

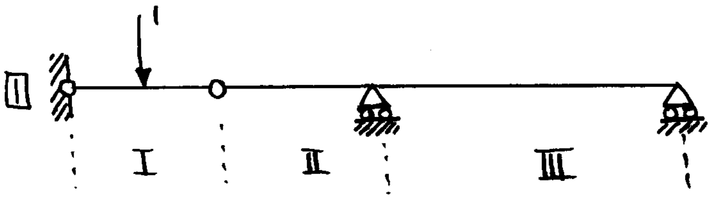
$$\Sigma M_c^i = 0 ; \quad M_{CD}^f + M_{CB}^2 + M_{CB}^3 + M_{CD}^3 = 0$$

$$8.33 + \frac{4EI}{10}\theta_B + \left(\frac{8EI}{10} + \frac{4EI}{10}\right)\theta_c = 0$$

$$\frac{EI}{10}\begin{bmatrix} 4 & 2 & 0 \\ 2 & 12 & 4 \\ 0 & 4 & 12 \end{bmatrix}\begin{bmatrix} \theta_A \\ \theta_B \\ \theta_c \end{bmatrix} = \begin{bmatrix} -12.5 \\ 12.5 \\ -8.33 \end{bmatrix} \longrightarrow \begin{bmatrix} \theta_A \\ \theta_B \\ \theta_c \end{bmatrix} = \begin{bmatrix} -\frac{42.38}{EI} \\ \frac{22.27}{EI} \\ -\frac{14.36}{EI} \end{bmatrix}$$

<문제 4>

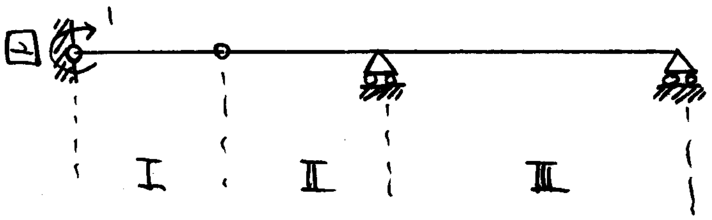
- Compatibility condition.



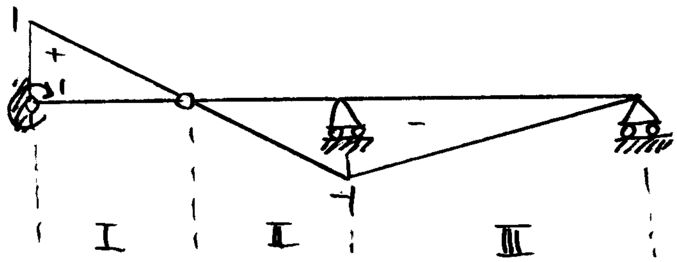
- compatibility condition
  (by Müller-Breslau)

$$d_{AX} + M_A \theta_{AA} = 0$$

$$M_A = -\frac{d_{AX}}{\theta_{AA}} = -\frac{d_{XA}}{\theta_{AA}}$$



- BMD (Bending Moment Diagram) for structure $\boxed{2}$



$$\therefore \theta_{AA} = \frac{1}{EI}\int_0^{\frac{L}{2}} \left(1-\frac{2}{L}x\right)^2 dx + \frac{1}{EI}\int_0^{\frac{L}{2}} \left(-\frac{2}{L}x\right)^2 dx + \frac{1}{EI}\int_0^{L} \left(-1+\frac{x}{L}\right)^2 dx = \boxed{\frac{2L}{3EI}}$$

- d_{XA}

1) span I

$$EI w_1'' = -\left(1-\frac{2}{L}x\right) \longrightarrow EI w_1 = -\frac{1}{2}x^2 + \frac{1}{3L}x^3 + ax + b$$

2) span II

$$EI w_2'' = -\left(-\frac{2}{L}x\right) \longrightarrow EI w_2 = \frac{1}{3L}x^3 + cx + d$$

3) span III

$$EI w_3'' = -\left(-1+\frac{x}{L}\right) \longrightarrow EI w_3 = \frac{1}{2}x^2 - \frac{1}{6L}x^3 + ex + f$$

- **Boundary Conditions**

1) $w_1(0) = 0$; $\underline{b = 0}$

2) $w_1\left(\frac{L}{2}\right) = w_2(0)$; $-\frac{1}{2}\left(\frac{L}{2}\right)^2 + \frac{1}{3L}\left(\frac{L}{2}\right)^3 + a\left(\frac{L}{2}\right) = d$

3) $w_2\left(\frac{L}{2}\right) = 0$; $\frac{1}{3L}\left(\frac{L^3}{8}\right) + c\left(\frac{L}{2}\right) + d = 0$

4) $w_3(0) = 0$; $\underline{f = 0}$

5) $w_3(L) = 0$; $\frac{1}{2}L^2 - \frac{1}{6L}L^3 + eL = 0 \rightarrow \underline{e = -\frac{L}{3}}$

6) $w_2'\left(\frac{L}{2}\right) = w_3'(0)$; $\frac{1}{L}\left(\frac{L}{2}\right)^2 + c = e \rightarrow \underline{c = -\frac{7}{12}L}$

$\rightarrow \underline{d = \frac{L^2}{4}}$ $\quad \underline{a = \frac{2L}{3}}$

- **Final Solution (Influence Line)**

1) Span I

$$M_A = -\frac{3}{2L}\left[\frac{1}{3L}x^3 - \frac{1}{2}x^2 + \frac{2L}{3}x\right]$$

2) Span II

$$M_A = -\frac{3}{2L}\left[\frac{1}{3L}x^3 - \frac{7L}{12}x + \frac{L^2}{4}\right]$$

3) Span III

$$M_A = -\frac{3}{2L}\left[-\frac{1}{6L}x^3 + \frac{1}{2}x^2 - \frac{L}{3}x\right]$$

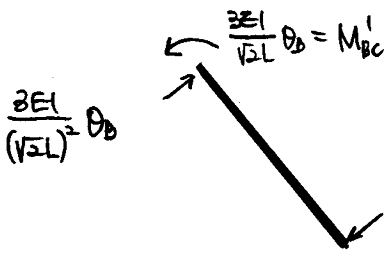
<문제 5>

- DOF : θ_B . Δ (→)

- Analysis

1) All fixed : no fixed end moment.

2) $\theta_B \neq 0$

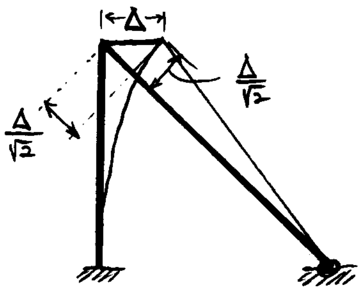


$$M_{AB}^1 = \frac{2EI}{L}\theta_B \qquad M_{BA}^1 = \frac{4EI}{L}\theta_B \qquad M_{BC}^1 = \frac{3EI}{\sqrt{2}L}\theta_B$$

$$V_{BA}^1 = \frac{6EI}{L^2}\theta_B$$

$$V_{BC}^1 = \frac{3EI}{2L^2}\theta_B \times \frac{1}{\sqrt{2}} = \frac{3EI}{2\sqrt{2}L^2}\theta_B$$

3) $\Delta \neq 0$



$$M_{AB}^2 = M_{BA}^2 = \frac{6EI}{L^2}\Delta \qquad V_{BA}^2 = \frac{12EI}{L^3}\Delta$$

$$M_{BC}^2 = \frac{3EI}{\sqrt{2}L} \times \frac{1}{\sqrt{2}L} \times \frac{\Delta}{\sqrt{2}} = \frac{3EI}{2\sqrt{2}L^2}\Delta$$

$$V_{BC}^2 = \frac{3EI}{4L^3}\Delta \times \frac{1}{\sqrt{2}} = \frac{3EI}{4\sqrt{2}L^3}\Delta$$

4) 사재의 축방향력.

$$A_{BC} = \frac{EA}{\sqrt{2}L} \times \frac{\Delta}{\sqrt{2}} = \frac{EA}{2L}\cdot\Delta \quad (\text{압축력})$$

$$V_{BC}^3 = \frac{EA}{2L}\cdot\Delta \times \frac{1}{\sqrt{2}} = \frac{EA}{2\sqrt{2}L}\Delta$$

- Stiffness Equation.

$\Sigma M_B^i = 0$; $\quad M_{BA}^1 + M_{BC}^1 + M_{BA}^2 + M_{BC}^2 = 0$

$$\left(4 + \frac{3}{\sqrt{2}}\right)\frac{EI}{L}\theta_B + \left(6 + \frac{3}{2\sqrt{2}}\right)\frac{EI}{L^2}\Delta = 0$$

$\Sigma V^i = 0$; $\quad V_{BA}^1 + V_{BC}^1 + V_{BA}^2 + V_{BC}^2 + V_{BC}^3 = P$

$$\left(6 + \frac{3}{2\sqrt{2}}\right)\frac{EI}{L^2}\theta_B + \left\{\left(12 + \frac{3}{4\sqrt{2}}\right)\frac{EI}{L^3} + \frac{EA}{2\sqrt{2}L}\right\}\Delta = P$$

<문제 6>

- Element Stiffness Matrix

$$k_{AB} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 12 & -6L & 0 & 0 \\ 6L & 2L^2 & -6L & 4L^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad k_{BC} = \frac{EI}{L^3}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 12 & 6L & -12 & 6L \\ 0 & 0 & 6L & 4L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$k_{spring} = \frac{EI}{L^3}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- Global Stiffness Matrix

$$K = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 13 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

- Stiffness Equation

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 13 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix}\begin{array}{l} \leftarrow \text{Known} \\ \leftarrow \text{Known} \\ \leftarrow \text{Known} \\ \leftarrow \text{Unknown} \\ \leftarrow \text{Unknown} \\ \leftarrow \text{Known} \end{array}$$

$$\begin{bmatrix} P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0 \\ -P \end{bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 8L^2 & -6L \\ -6L & 13 \end{bmatrix}\begin{bmatrix} u_4 \\ u_5 \end{bmatrix}$$

$$\begin{bmatrix} u_4 \\ u_5 \end{bmatrix} = \frac{L^3}{EI}\begin{bmatrix} 8L^2 & -6L \\ -6L & 13 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ P \end{bmatrix} = \begin{bmatrix} -\frac{3PL^2}{34EI} \\ -\frac{2PL^3}{17EI} \end{bmatrix} = \begin{bmatrix} \theta_B\ (\circlearrowleft) \\ w_c\ (\uparrow) \end{bmatrix}$$

• Reactions

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_6 \end{bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 6L & 0 \\ 2L^2 & 0 \\ 0 & -12 \\ 2L & -6L \end{bmatrix}\begin{bmatrix} u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} -\frac{9}{17}P \\ -\frac{3}{17}PL \\ \frac{24}{17}P \\ \frac{9}{17}PL \end{bmatrix} = \begin{bmatrix} R_A\ (\uparrow) \\ M_A\ (\circlearrowleft) \\ R_B\ (\uparrow) \\ M_c\ (\circlearrowleft) \end{bmatrix}$$

• 각 자유도에서의 처짐 ($\uparrow\oplus$, $\circlearrowleft\oplus$)

$$\begin{bmatrix} w_A \\ \theta_A \\ w_B \\ \theta_B \\ w_c \\ \theta_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{3PL^2}{34EI} \\ -\frac{2PL^3}{17EI} \\ 0 \end{bmatrix}$$

• 각 지점에서 발생하는 반력

$$\begin{bmatrix} R_A \\ M_A \\ R_B \\ M_B \\ R_c \\ M_c \end{bmatrix} = \begin{bmatrix} -\frac{9}{17}P \\ -\frac{3}{17}PL \\ \frac{24}{17}P \\ 0 \\ -P \\ \frac{9}{17}PL \end{bmatrix}$$