

1. (Euler operator)

- (a) MEV (B, L1, V2, &E2, &V3, -D/2, -W/2, 0)
- (b) MEV (B, L1, V1, &E5, &V5, D/2, W/2, H)
- (c) MEL (B, L1, V5, V6, &E9, &L3)

2. (B-spline curve)

(a) - Number of knot value: $n + k + 1 = 5 + 4 + 1 = 10$

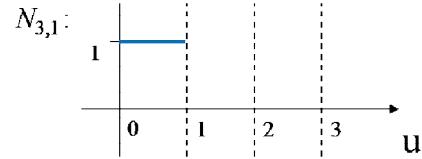
- Knot value: $t_0 = t_1 = t_2 = t_3 = 0, t_4 = 1, t_5 = 2, t_6 = t_7 = t_8 = t_9 = 3$

(b) From the knot values, it can be found that the range of parameter u is $0 \leq u \leq 3$ and the curve is composed of 3 different B-spline curves combined at $u = 1, 2$.

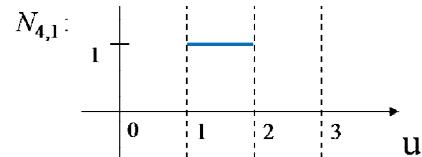
(c) – Calculation of blending function:

For $k = 1$,

$$N_{3,1} = \begin{cases} 1 & (0 \leq u < 1) \\ 0 & (\text{other region}) \end{cases}$$



$$N_{4,1} = \begin{cases} 1 & (1 \leq u < 2) \\ 0 & (\text{other region}) \end{cases}$$

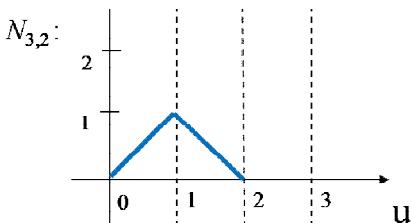


For $k = 2$,

$$N_{3,2}(u) = \frac{u - t_3}{t_4 - t_3} \cdot N_{3,1}(u) + \frac{t_5 - u}{t_5 - t_4} \cdot N_{4,1}(u)$$

$$= uN_{3,1}(u) + (2 - u)N_{4,1}(u)$$

$$= \begin{cases} u & (0 \leq u \leq 1) \\ 2 - u & (1 \leq u \leq 2) \\ 0 & (\text{other region}) \end{cases}$$



3. (Differentiation of a Bezier curve equation)

$$\mathbf{P}(u) = \sum_{i=0}^n \binom{n}{i} u^i (1-u)^{n-i} \mathbf{P}_i \quad (0 \leq u \leq 1)$$

Differentiate this equation with respect to u

$$\begin{aligned} \frac{d\mathbf{P}(u)}{du} &= \sum_{i=0}^n i \binom{n}{i} u^{i-1} (1-u)^{n-i} \mathbf{P}_i - \sum_{i=0}^n (n-i) \binom{n}{i} u^i (1-u)^{n-i-1} \mathbf{P}_i \\ &= \sum_{i=1}^n i \binom{n}{i} u^{i-1} (1-u)^{n-i} \mathbf{P}_i - \sum_{i=0}^{n-1} (n-i) \binom{n}{i} u^i (1-u)^{n-i-1} \mathbf{P}_i \end{aligned}$$

Substitute $j = i - 1$

$$= \sum_{j=0}^{n-1} (j+1) \binom{n}{j+1} u^j (1-u)^{n-j-1} \mathbf{P}_{j+1} - \sum_{i=0}^{n-1} (n-i) \binom{n}{i} u^i (1-u)^{n-i-1} \mathbf{P}_i$$

The terms $(j+1) \binom{n}{j+1}$ and $(n-i) \binom{n}{i}$ can be expanded as follows

$$(j+1) \binom{n}{j+1} = \frac{(j+1)n!}{(j+1)! (n-j-1)!} = \frac{n(n-1)!}{j! (n-j-1)!} = n \binom{n-1}{j}$$

$$(n-i) \binom{n}{i} = \frac{(n-i)n!}{i! (n-i)!} = \frac{n(n-i)!}{i! (n-i-1)!} = n \binom{n-1}{i}$$

Substituting equations

$$\begin{aligned} \frac{d\mathbf{P}(u)}{du} &= \sum_{j=0}^{n-1} n \binom{n-1}{j} u^j (1-u)^{n-j-1} \mathbf{P}_{j+1} - \sum_{i=0}^{n-1} n \binom{n-1}{i} u^i (1-u)^{n-i-1} \mathbf{P}_i \\ &= n \sum_{i=0}^{n-1} \binom{n-1}{i} u^i (1-u)^{n-i-1} (\mathbf{P}_{i+1} - \mathbf{P}_i) \end{aligned}$$

Therefore,

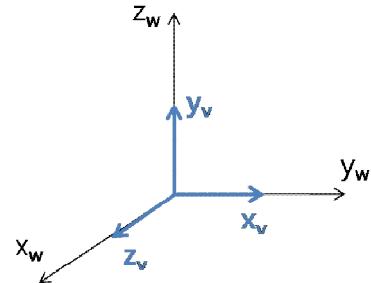
$$\frac{d\mathbf{P}(u)}{du} = n \sum_{i=0}^{n-1} \binom{n-1}{i} u^i (1-u)^{n-1-i} \mathbf{a}_i$$

where $\mathbf{a}_i = \mathbf{P}_{i+1} - \mathbf{P}_i \quad i = 0, 1, \dots, n-1$

4. (Matrix transformation, mapping)

view point: (10, 0, 0), view site: (0, 0, 0), up vector: (0, 0, 1)

$$T_{w-v} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



(n_x, n_y, n_z) are $(0, 0, 1)$ because they are the x_v, y_v, z_v components of the x_w axis.

(o_x, o_y, o_z) are $(1, 0, 0)$ because they are the x_v, y_v, z_v components of the y_w axis.

(a_x, a_y, a_z) are $(0, 1, 0)$ because they are the x_v, y_v, z_v components of the z_w axis.

(p_x, p_y, p_z) are $(0, 0, 0)$ because they are the x_v, y_v, z_v coordinate of the origin of the x_w, y_w, z_w coordinate system.

Therefore,

$$T_{w-v} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. (OpenGL)

(a) [GL_QUAD_STRIP](#)

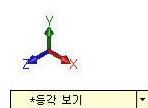
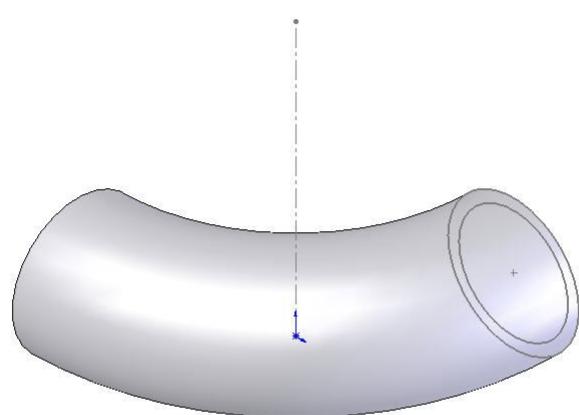
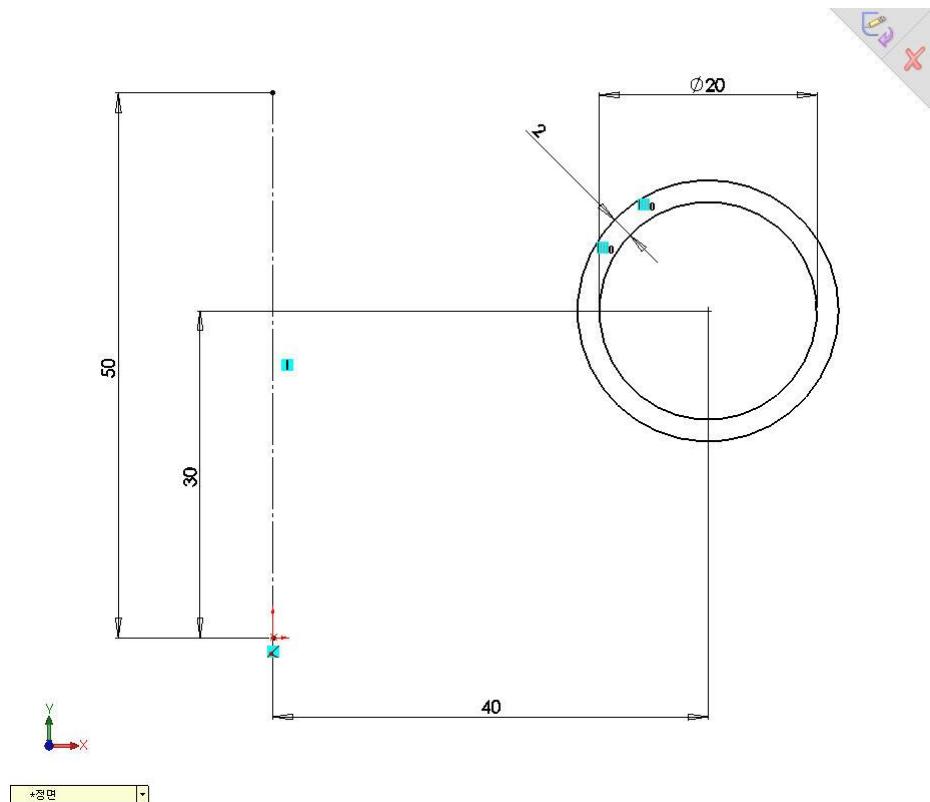
(b) $a * \cos(\text{Phi}) * \cos(\text{Theta})$
 $b * \cos(\text{Phi}) * \sin(\text{Theta})$
 $c * \sin(\text{Phi})$
 $a * \cos(\text{Phi} + \text{step_Phi}) * \cos(\text{Theta})$
 $b * \cos(\text{Phi} + \text{step_Phi}) * \sin(\text{Theta})$
 $c * \sin(\text{Phi} + \text{step_Phi})$

(c) [180.0, 1, 0, 0 or 180.0, 0, 1, 0](#)

6. (SolidWorks API)

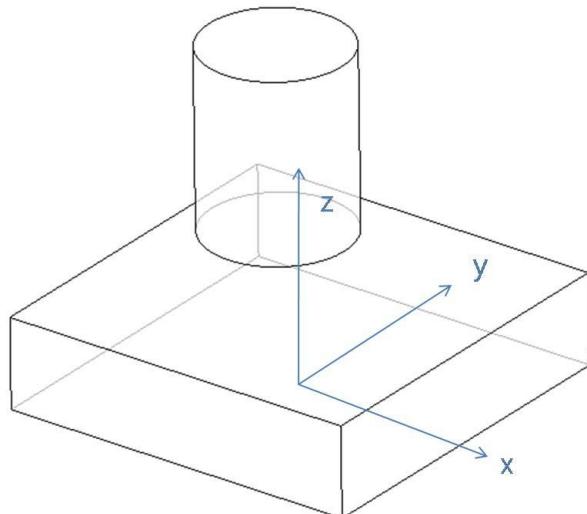
(a) swModel.CreateCircleByRadius2 0.04, 0.03, 0, 0.01

(b)

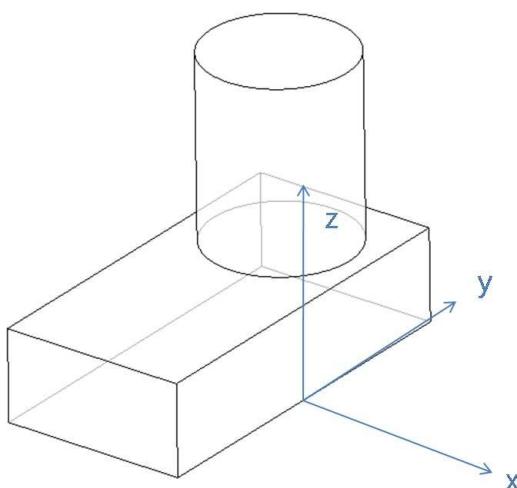


7. (Parasolid API)

(a)



(b)



(c) This function returns the partition to the state when the given pmark was created.

