

(1.)

a) linearity

$$y = ay_1 + by_2$$

① superposition.

$$f(x+y) = f(x) + f(y) \Rightarrow (5)$$

② Homogeneity

$$f(\alpha x) = \alpha f(x) \text{ (for all } \alpha) \Rightarrow (5)$$

b) the order of a dynamic system.

m, k,  $\frac{b}{m}$ , energy storage elements <sup>number</sup> ~~number~~  $\Rightarrow (10)$

c)

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0.$$

$$\omega_n = \sqrt{\frac{k}{m}}, \quad 2\zeta\omega_n = \frac{b}{m} \Rightarrow (5)$$

$$\zeta = \frac{b}{2m} \sqrt{\frac{m}{k}} = \frac{b}{2\sqrt{mk}} \Rightarrow (5)$$

$$2. \quad \dot{y}_1 = -2y_2 - y_1 + e^{-t} \quad \dots (1)$$

$y_2$   
F

$y_1$   
—

$$\dot{y}_2 = y_1 \quad \dots (2)$$

A. Put (2) to (1),

$$\ddot{y}_2 = -2y_2 - \dot{y}_2 + e^{-t}$$

$$\Rightarrow \ddot{y}_2 + \dot{y}_2 + 2y_2 = e^{-t}$$

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B. Let the input of the system is  $x(t) = e^{-t}$ ,

$$\ddot{y}_2 + \dot{y}_2 + 2y_2 = x(t) \xrightarrow{\mathcal{L}} (s^2 + s + 2)Y_2(s) = X(s)$$

$$\frac{Y_2(s)}{X(s)} = \frac{1}{s^2 + s + 2} \quad \dots \text{transfer function}$$

$$x(t) = e^{-t} \xrightarrow{\mathcal{L}} X(s) = \frac{1}{s+1} \quad \dots \text{Input}$$

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$$C. \quad Y_2(s) = \frac{1}{s^2 + s + 2} \cdot \frac{1}{s+1} \quad \dots (a)$$

$$(a); \quad \frac{As+B}{s^2 + s + 2} + \frac{C}{s+1} = \frac{1}{(s^2 + s + 2)(s+1)}$$

$$\Rightarrow (As+B)(s+1) + C(s^2 + s + 2) = (A+C)s^2 + (A+B+C)s + B+2C = 1$$

$$\therefore \left. \begin{aligned} A+C &= 0 \\ A+B+C &= 0 \\ B+2C &= 1 \end{aligned} \right\} \begin{aligned} B &= 0 \\ C &= \frac{1}{2} \\ A &= -\frac{1}{2} \end{aligned}$$

$$2. Y_2(s) = \frac{1}{2} \left[ \frac{1}{s+1} - \frac{s}{s^2+s+2} \right]$$

$$= \frac{1}{2} \left[ \frac{1}{s+1} - \frac{(s+\frac{1}{2}) - \frac{1}{2}}{(s+\frac{1}{2})^2 + \frac{7}{4}} \right]$$

$$= \frac{1}{2} \left[ \frac{1}{s+1} - \frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2 + \frac{7}{4}} + \frac{1}{2} \cdot \frac{1}{(s+\frac{1}{2})^2 + \frac{7}{4}} \right]$$

$$= \frac{1}{2} \left[ \frac{1}{s+1} - \frac{(s+\frac{1}{2})}{(s+\frac{1}{2})^2 + (\frac{\sqrt{7}}{4})^2} + \frac{1}{2} \cdot \frac{\sqrt{7}}{4} \cdot \frac{\sqrt{7}}{(s+\frac{1}{2})^2 + (\frac{\sqrt{7}}{4})^2} \right]$$

$$= \frac{1}{2} \left[ \frac{1}{s+1} - \frac{(s+\frac{1}{2})}{(s+\frac{1}{2})^2 + (\frac{\sqrt{7}}{4})^2} + \frac{1}{\sqrt{7}} \cdot \frac{\sqrt{7}}{(s+\frac{1}{2})^2 + (\frac{\sqrt{7}}{4})^2} \right]$$

$$y_2(t) = \mathcal{L}^{-1}\{Y_2(s)\}$$

$$= \frac{1}{2} \left[ e^{-t} - e^{-\frac{1}{2}t} \cdot \cos \frac{\sqrt{7}}{2} t + \frac{\sqrt{7}}{7} e^{-\frac{1}{2}t} \cdot \sin \frac{\sqrt{7}}{2} t \right]$$

$$= \frac{1}{2} e^{-t} - \frac{1}{2} e^{-\frac{1}{2}t} \cos \frac{\sqrt{7}}{2} t + \frac{\sqrt{7}}{14} e^{-\frac{1}{2}t} \cdot \sin \frac{\sqrt{7}}{2} t$$

$$y_1(t) = \frac{d}{dt}(y_2(t))$$

$$= -\frac{1}{2} e^{-t} + \frac{1}{4} e^{-\frac{1}{2}t} \cos \frac{\sqrt{7}}{2} t + \frac{\sqrt{7}}{4} e^{-\frac{1}{2}t} \cdot \sin \frac{\sqrt{7}}{2} t$$

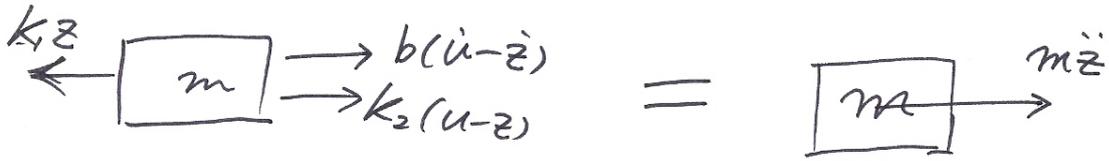
$$- \frac{\sqrt{7}}{28} e^{-\frac{1}{2}t} \sin \frac{\sqrt{7}}{2} t + \frac{7}{28} e^{-\frac{1}{2}t} \cos \frac{\sqrt{7}}{2} t$$

$$= -\frac{1}{2} e^{-t} + \frac{1}{2} e^{-\frac{1}{2}t} \cos \frac{\sqrt{7}}{2} t + \frac{3\sqrt{7}}{14} e^{-\frac{1}{2}t} \sin \frac{\sqrt{7}}{2} t$$

//  
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(3)

F.B.D



$$m \ddot{z} + b \dot{z} + (k_1 + k_2) z = b \dot{u} + k_2 u$$

$\Rightarrow$  (5)

$$\mathcal{L} \Rightarrow \frac{Z}{U} = \frac{bs + k_2}{ms^2 + bs + (k_1 + k_2)}$$

$$\left\{ \begin{aligned} \frac{X}{U} &= \frac{1}{ms^2 + bs + (k_1 + k_2)} \Rightarrow m \ddot{x} + b \dot{x} + (k_1 + k_2)x = u \\ \frac{Z}{X} &= bs + k_2 \Rightarrow b \dot{x} + k_2 x = z \end{aligned} \right.$$

bet.  $\begin{cases} x_1 = x \\ x_2 = \dot{x} \end{cases} \Rightarrow \begin{cases} \dot{x}_1 = \dot{x} = x_2 \\ \dot{x}_2 = \ddot{x} = -\frac{b}{m} \dot{x} - \frac{(k_1 + k_2)}{m} x + \frac{1}{m} u \end{cases}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k_1 + k_2}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

$$z = \begin{bmatrix} k_2 & b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$\Rightarrow$  (10)

다른 방법

$$\frac{d}{dt} \left( \dot{z} - \frac{b}{m} u \right) = -\frac{b}{m} \left( \dot{z} - \frac{b}{m} u \right) - \left( \frac{b}{m} \right)^2 u + \frac{K_2}{m} u - \frac{(K_1 + K_2)}{m} z$$

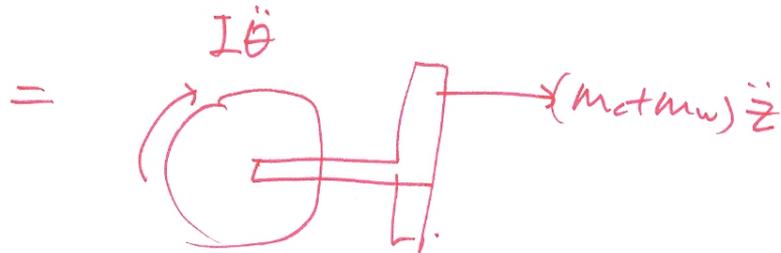
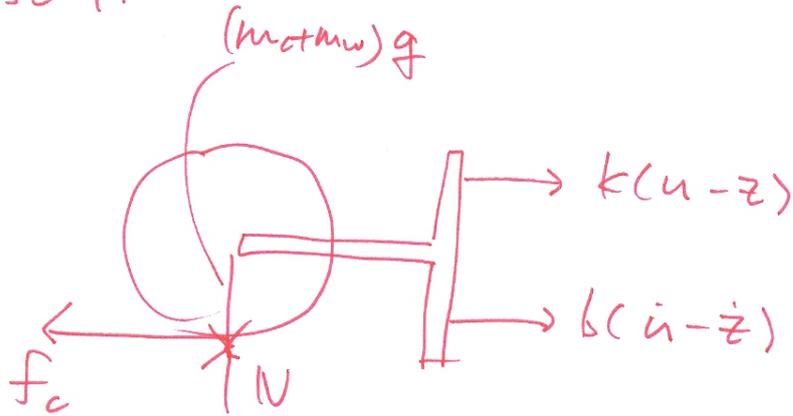
$$\begin{cases} x_1 = z \\ x_2 = \dot{z} - \frac{b}{m} u \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{(K_1 + K_2)}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{b}{m} \\ -\left(\frac{b}{m}\right)^2 + \frac{K_2}{m} \end{bmatrix} u.$$

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4. a

case 1.



$m_c g$   
 $m_w g$   
 $I \ddot{\theta}$   
 $m_c \ddot{z}$   
 $m_w \ddot{z}$

} 각 1점.

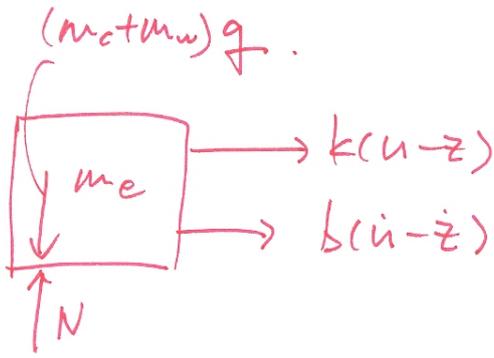
$f_c$  : 2점.

$k(u - z)$   
 $b(\dot{u} - \dot{z})$

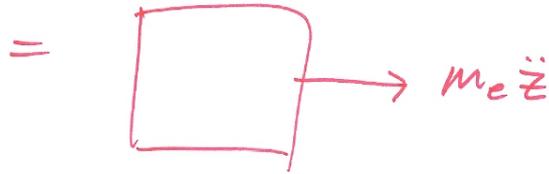
} 하나쓰면 1점  
 둘다쓰면 3점.

총 10점.

Case 2.



$$m_e = \frac{3}{2}m_w + m_c$$



$m_c g$   
 $m_w g$ 
} → 각 1점.

$m_e = \frac{3}{2}m_w + m_c$  : 2점.

$m_e \ddot{z}$  : 3점.

$k(u - z)$   
 $b(\dot{u} - \dot{z})$ 
} → 하나 쓰면 1점  
 둘 다 쓰면 3점.

b. case 1.

$$m_c \ddot{z} = b(\dot{u} - \dot{z}) + k(u - z) - F$$

$$m_w \ddot{z} = -f_c + F$$

→ 둘다 쓰거나 하나의 식으로 쓰면 3점  
하나만 맞으면 1점.

$$\frac{1}{2} m_w \omega^2 = I_w \quad : \quad 1 \text{ 점.}$$

$$I \ddot{\theta} = r f_c \quad : \quad 2 \text{ 점.}$$

$$r \theta = z \quad : \quad 3 \text{ 점.}$$

답이 맞으면 1점

총 10점.

case 2.

$m_{eq}$  구하는 과정 : 3점

$$m_{eq} \ddot{z} = b(\dot{u} - \dot{z}) + k(u - z) \quad : \quad 3 \text{ 점.}$$

$$r \theta = z \quad : \quad 3 \text{ 점.}$$

답이 맞으면 1점

총 10점.

다음

$$\left( \frac{3}{2} m_w + m_c \right) \ddot{z} + b \dot{z} + k z = b \dot{u} + z u.$$

C.

b가 틀려도.

b에서 구한 답으로 TF를 구하면 3점.

b의 답이 맞고 TF도 맞으면 5점.

$$\text{답 TF} = \frac{bs + k}{\left(\frac{3}{5}m_w + m_c\right)s^2 + bs + k} \quad \text{총 5점.}$$

D.

$$f_c = \mu_s (m_w + m_c) g \quad : 2\text{점.}$$

~~m\_c~~  $\rightarrow$   $m_c$ 를 빼먹으면 1점.

$$r f_c < I \ddot{\theta}$$

$$r \ddot{\theta} < \ddot{z}$$

등등의 조건을 써서 풀어서 답이 맞으면 3점.

부등호 틀리면 -2.

부등호에 대한 언급이 없어도 -2.

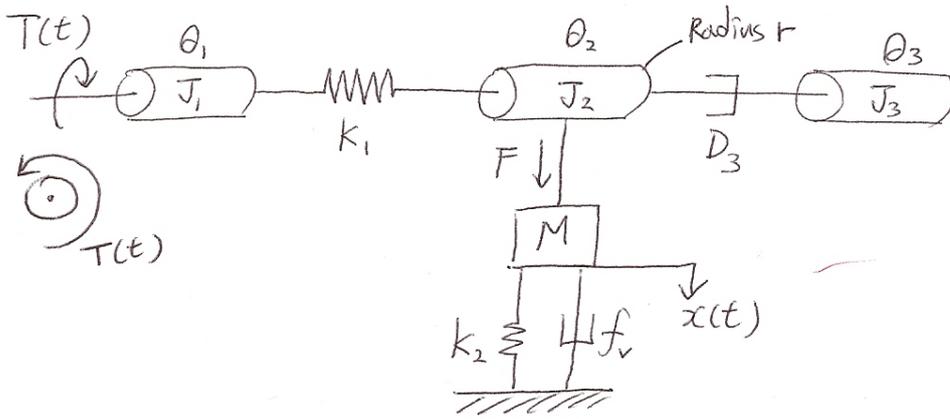
등호를 사용하고 slip이 일어나는 점이라는 말이 있으면 -1.

기타 계산 실수 -2.

총 5점.

답:  $\ddot{z} > 2 \frac{m_w + m_c}{m_w} \mu_s g$  (정리가 안되어있어도 앞과정으로 미루어 맞으면 정답. 다음부터 정리 해주시길)

5. <FBD>



< Modeling equation > ... each 3pt (22/6 1pt, 4 2pt)

① ;  $J_1 \ddot{\theta}_1 + k_1(\theta_1 - \theta_2) = T(t) \dots (1)$

② ;  $J_2 \ddot{\theta}_2 + D_3(\dot{\theta}_2 - \dot{\theta}_3) + k_1(\theta_2 - \theta_1) + F \cdot r = 0 \dots (2)$

③ ;  $J_3 \ddot{\theta}_3 + D_3(\dot{\theta}_3 - \dot{\theta}_2) = 0 \dots (3)$

④ ;  $M \ddot{x} + f_v \dot{x} + k_2 x = F \dots (4)$

< Geometry equation >

As gear ratio is 1=1

$$r \theta_2 = x$$

$$\Rightarrow r \dot{\theta}_2 = \dot{x}, r \ddot{\theta}_2 = \ddot{x}$$

// 3pt

15pt

Laplace transform **5 pt**

$$(1) ; (J_1 s^2 + k_1) \theta_1(s) - k_1 \theta_2(s) = T(s) \quad \dots (1)'$$

$$(2) : (J_2 s^2 + D_3 s + k_1) \theta_2(s) - k_1 \theta_1(s) - D_3 s \theta_3(s) + F(s) \cdot r = 0 \quad \dots (2)'$$

$$(3) : (J_3 s^2 + D_3 s) \theta_3(s) - D_3 s \theta_2(s) = 0 \quad \dots (3)'$$

$$(4) : (M s^2 + f_v s + k_2) X(s) = F(s) \quad \dots (4)'$$

from (1)', 
$$\theta_1(s) = \frac{k_1 \theta_2(s) + T(s)}{J_1 s^2 + k_1} \quad \dots (a)$$

from (3)', 
$$\theta_3(s) = \frac{D_3 s \cdot \theta_2(s)}{J_3 s^2 + D_3 s} \quad \dots (b)$$

put (a), (b), (4) in Eqn. (2)'

$$(J_2 s^2 + D_3 s + k_1) \theta_2(s) - \frac{k_1 (k_1 \theta_2(s) + T(s))}{J_1 s^2 + k_1} - \frac{D_3^2 s^2 \theta_2(s)}{J_3 s^2 + D_3 s} + r (M s^2 + f_v s + k_2) X(s) = 0$$

$$r \theta_2 = X \quad \xrightarrow{L} \quad r \theta_2(s) = X(s)$$

**5 pt**

$$\left[ (J_2 s^2 + D_3 s + k_1) - \frac{k_1^2}{J_1 s^2 + k_1} - \frac{D_3^2 s^2}{J_3 s^2 + D_3 s} + r^2 (M s^2 + f_v s + k_2) \right] \theta_2(s) = \frac{k_1}{J_1 s^2 + k_1} T(s)$$

$$\Rightarrow \underbrace{\left[ (J_2 + M r^2) s^2 + (D_3 + f_v r^2) s + (k_1 + k_2 r^2) \right]}_A - \frac{k_1^2}{J_1 s^2 + k_1} - \frac{D_3^2 s^2}{J_3 s^2 + D_3 s} \theta_2(s) = \frac{k_1}{J_1 s^2 + k_1} T(s)$$

$$\Rightarrow \underbrace{\left[ A (J_1 s^2 + k_1) (J_3 s^2 + D_3 s) - k_1^2 (J_3 s^2 + D_3 s) - D_3^2 s^2 (J_1 s^2 + k_1) \right]}_B \theta_2(s) = k_1 (J_3 s^2 + D_3 s) T(s)$$

$$\Rightarrow B \theta_2(s) = k_1 (J_3 s^2 + D_3 s) T(s)$$

**30 pt**

$$\Rightarrow B \cdot \frac{X(s)}{r} = k_1 (J_3 s^2 + D_3 s) T(s) \quad \Rightarrow \quad \frac{X(s)}{T(s)} = \frac{r k_1 (J_3 s^2 + D_3 s)}{B}$$

**5 pt**