Mechanical and Aerospace System Analysis

Final Exam

June. 9. 2009

1. Answer the following questions.
   A. Derive the frequency response of a system with a general transfer function G(s). (10점)

   B. For a second order system with a transfer function (10점)

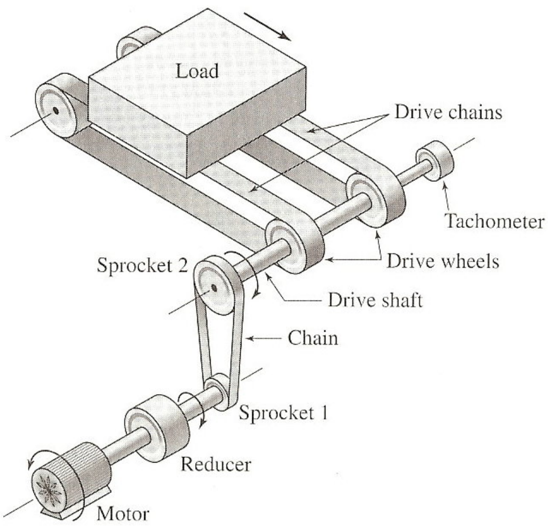
   $$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

      i. Derive the magnitude of the frequency response as a function of the frequency w.
      ii. Derive the phase of the frequency response as a function of the frequency w.

   C. If you are given a frequency response plot of a second order system, (5점)
      i. How would you find the natural frequency?
      ii. How would you find the damping ratio of the system?

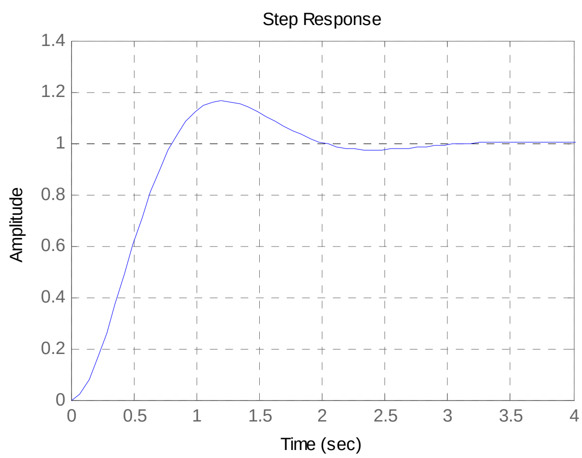
   D. For a system described with a state space equation $\dot{\mathrm{x}} = \mathrm{A}\,\mathrm{x} + \mathrm{B}u$,
      i. Derive the State Transition Matrix by finding the solution of the equation. (10점)
      ii. Write 5 properties of the state transition matrix (5점)

2. Conveyor Drive System to produce translation of the load is shown in the figure. To translate the load a specified distance, the drive wheels must rotate through a required angle and this can be accomplished by controlling the speed, often with a trapezoidal speed profile. The gear ratio of the reducer is 1:5 and the gear ratio between sprocket 1 and 2 is 1:2. Sprocket 2 and the drive wheels have same diameter. The load is 100 kg. The inertia of the drive wheels and the conveyor system is 1 kg m^2. The diameter of the drive wheel is 0.2 meter. Assume the system is rigid and the damping in the system is negligible. There is a coulomb friction of T_F at the motor shaft. The motor has an inductance of 1mH and resistance of 1 Ohm in series. The back emf constant K_b is 0.01 V/rad/s and the torque constant K_T is 0.1 Nm/Amp.



A. Draw the F.B.D of the system. If you need to, divide the system into components. (5점)
B. Derive the equivalent inertia of the load and all the drive components felt at the motor shaft I_e. (10점)
C. Derive the equation of motion of the drive system. (5점)
D. Derive the equation of the motion of the motor. (5점)
E. Derive the transfer function of the whole system with the current as the input and the speed of the motor as the output. (5점)
F. Draw a transient response of the system. (10점)
   i. Mark the peak time and the settling time and the % overshoot.
G. Draw the bode plot of the system. (10점)
   i. Mark the frequency where the peak occurs, and where the phase changes 90 degrees.
H. If the load is changed to 200kg, how will the bode plot change? (Describe qualitatively, in terms of change in the frequencies where the peak occurs, and where the phase of 90 degrees occurs.) (5점)

3. Following is the step response of a second order system.



From the graph, the peak time is 1.2 secs, the settling time is 2.66 secs.

i. Find the natural frequency and damping ratio of the system. Round off the values to the first digit after decimal. (10점)
ii. Draw a bode plot of the system. Make sure to plot the following.(10점)
   A. Magnitude of the frequency response at initial frequency (near 0 Hz)
   B. The frequency where the peak M_m occurs, W_m.
   C. The overshoot of the frequency response plot.

D. The frequency where the phase plot crosses 90 degrees.

E. Slope of the frequency response plot.

iii. Write the transfer function of the system. (5점)

iv. Draw the pole zero map of the system (5점)

v. If we want to reduce the peak time in half without changing the overshoot of the system, (10점)

A. Where should the poles move to?

B. How does the frequency response plot change?

4. Following shows frequency response of four different systems. (20점)

Plot step responses of the four systems in one figure. (Show the relative differences in peak time, % overshoot and settling time of the four systems. The graph does not have to be exact as long as the relative differences are shown clearly.)

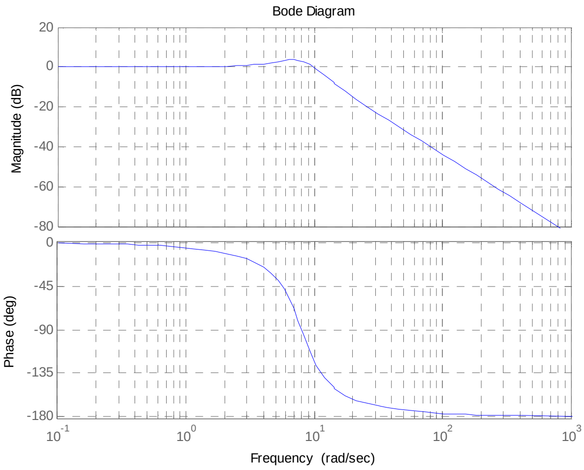


Fig. A

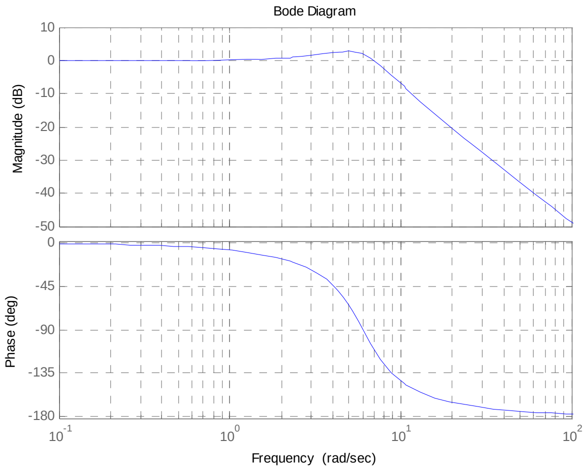


Fig. B

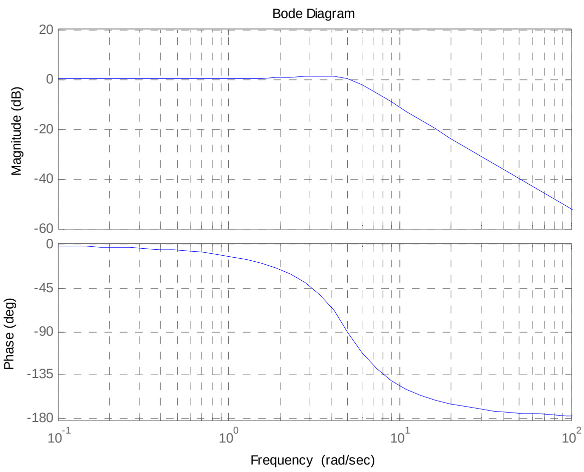


Fig. C

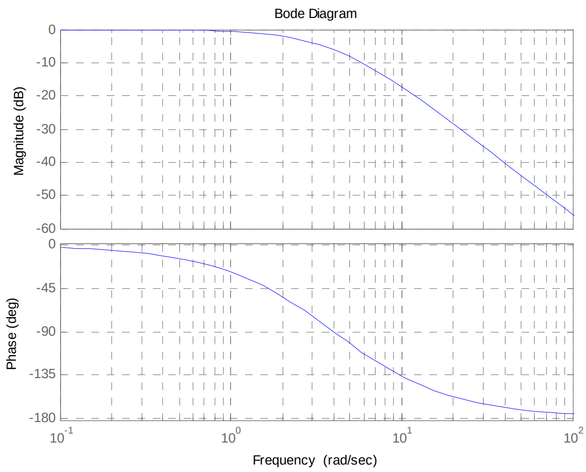


Fig. D